Digital Systems

Week 2: Number Systems and Boolean Algebra

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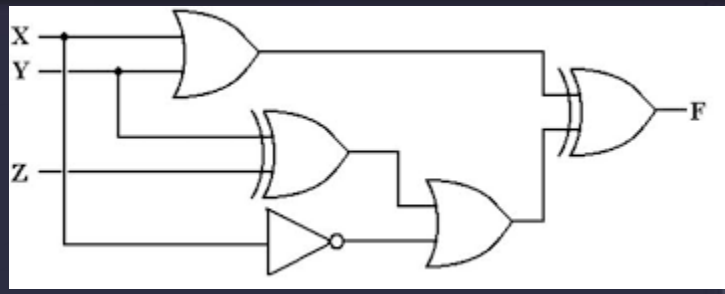
Course Plan

- Number Systems
- Boolean Algebra

Why Digital Systems?

- Computer Hardware
  - Softwares that require performance can be only written by who have a deep understanding of hardware.

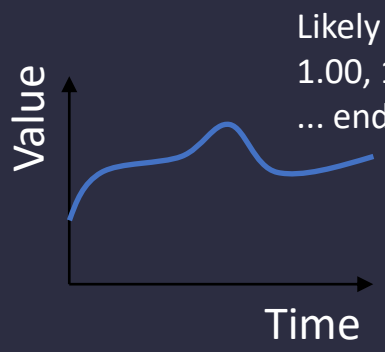
- Almost all electronic devices are digital
    - Audio recorders , cameras , vehicles ph1s , medical devices…
    - Developing equipment needed in almost every industry
  - It is an area that is highly needed both in our country and abroad. It could be a different career goal for you.



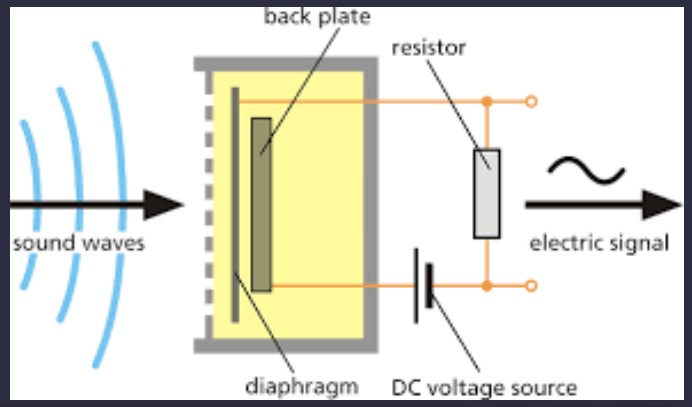
What Does Digital Mean?

- Analog Signal
  - It has infinite possible value.
    - For example, the vibration created by a microph1 on the line.

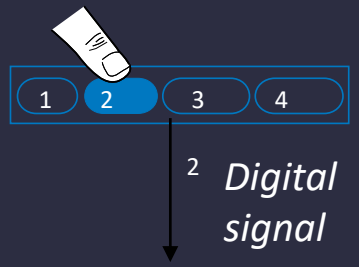
Analog Signal

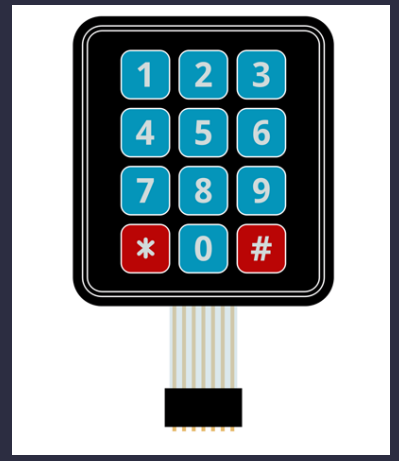


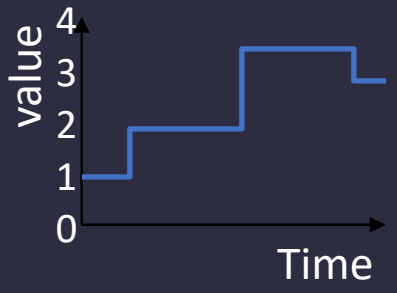
Likely Values
1.00, 1.01, 20000009,
... endless possibilities



- Digital Signal
  - Finite possible values
    - For example : Pressing a button on a keypad





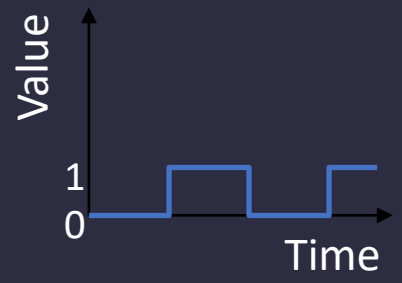


Possible values :
0, 1, 2, 3, or 4.

There are no other possible values

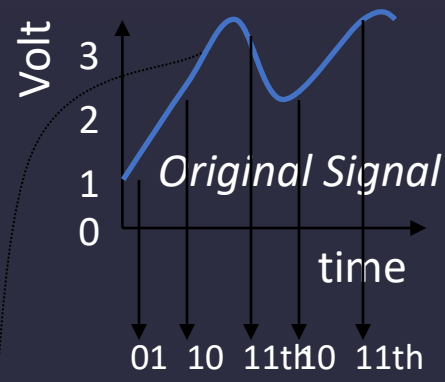
Digital Signals with Only Two Values: Binary

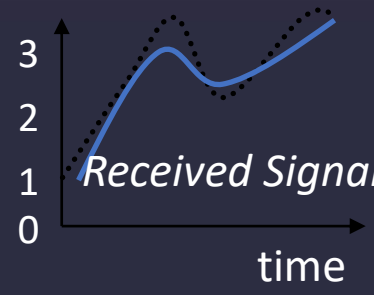
- ***Binary*** digital signals have only two possible values
  - These are shown as 0 and 1
  - A binary digit is expressed as a "bit".
  - Within the scope of the course, binary digital systems will be considered.
  - Binary is popular because:
    - Transistors , the most basic digital electrical comp1nt , operate at two voltage values (0 and 1)



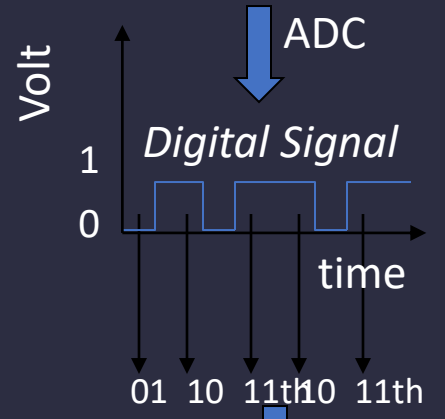
Advantages of Digitization

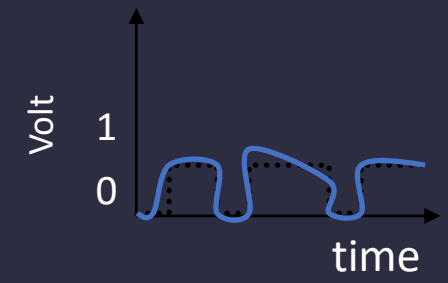
- Analogue signal is very sensitive to noise
  - During transmission, voltage levels may change due to many factors.
- Digital signals are more resistant to degradation during transmission.
  - Voltage levels still may not transmit perfectly
  - However, some distorted 1s and 0s can be recovered.



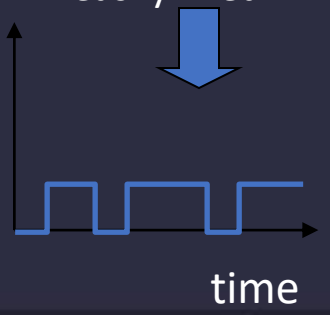


How can it be fixed???





Distorted 1s and 0s can be easily fixed



Digitized Content, Compression Benefits

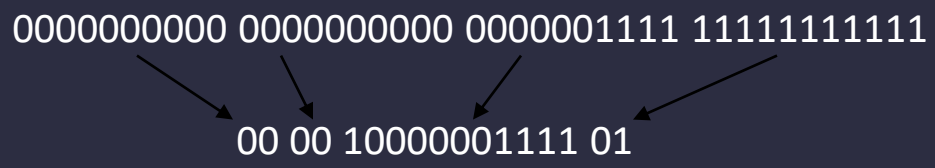
- Digitized Audios can be compressed
  - eg . MP3
- Compression can also be d1 on photos (jpeg) or videos (mpeg)
- Digitization has many different advantages.

Example Compression Table

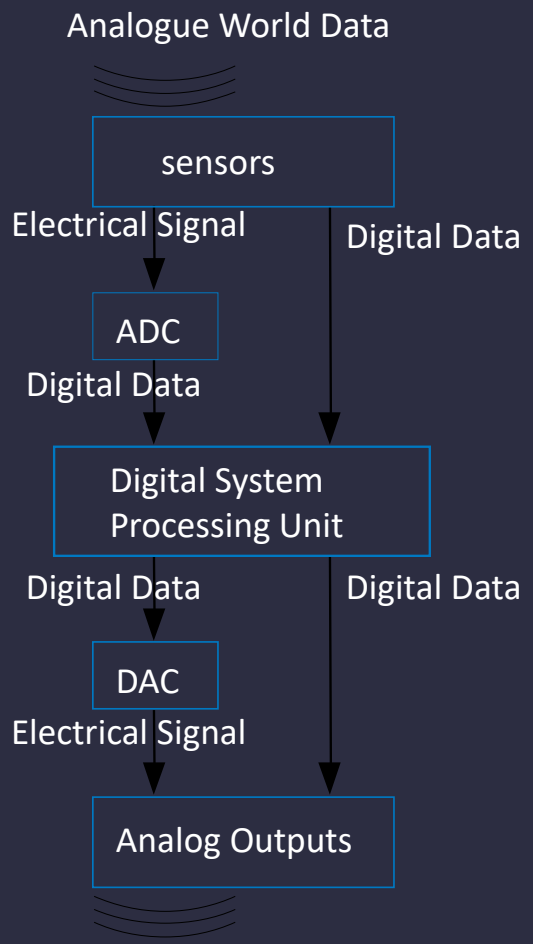
00 --> 0000000000

01 --> 11111111111

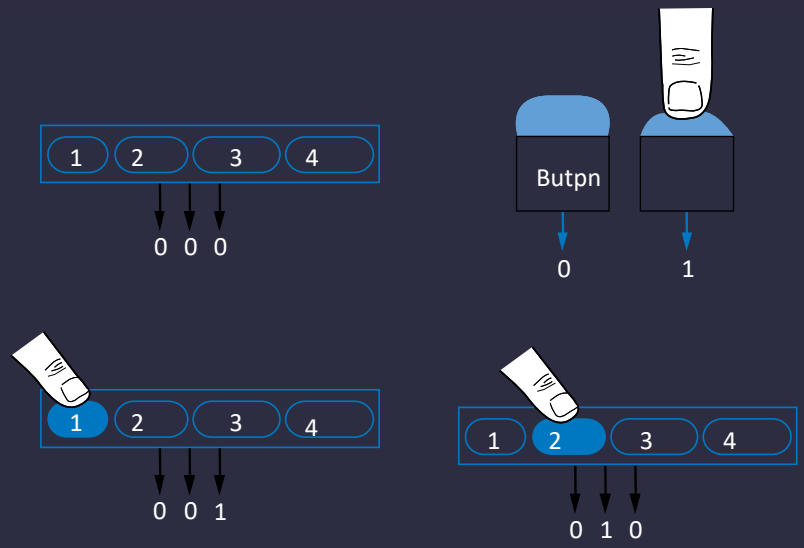
1X --> X

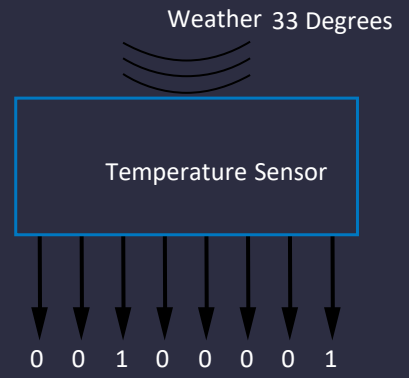


Binary Data Encode



- If button is not pressed (0), if pressed (1)
- Multi-button : coding
  1st button =001, 2nd button =010, ...
- Some inputs are analog
  - Requires an analog-to-digital converter to switch to digital.
- ADC (Analog to Digital Converter): Converts analog signal to digital
- DAC (Digital to Analog Converter): Converts digital signal to analog





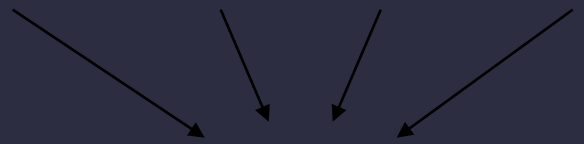
ASCII Encoding

- ASCII: 8 bits of each character and symbol. It is a table with the corresponding

Symbol	Machine code
R	1010010
S	1010011
T	1010100
L	1001100
N	1001110
TO	1000101
0	0110000
.	0101110
<tab>	0001001

Symbol	Machine code
r	1110010
s	1110011
t	1110100
l	1101100
n	1101110
to	1100101
9	0111001
!	0100001
<space>	0100000

1010010 1000101 1010011 1010100



REST

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

Numbers Encoding

- Decimal base (***decimal***)
  - There are 10 symbols : 0, 1, 2, ..., 8, and 9
  - After 9 comes a new digit
    - So each digit is a power of 10.
    - Base of 10 is used as it is suitable for daily life operations.
- Binary Base (***binary***)
  - There are two symbols : 0 and 1
  - New power comes after 1
    - So each digit is a power of 2.

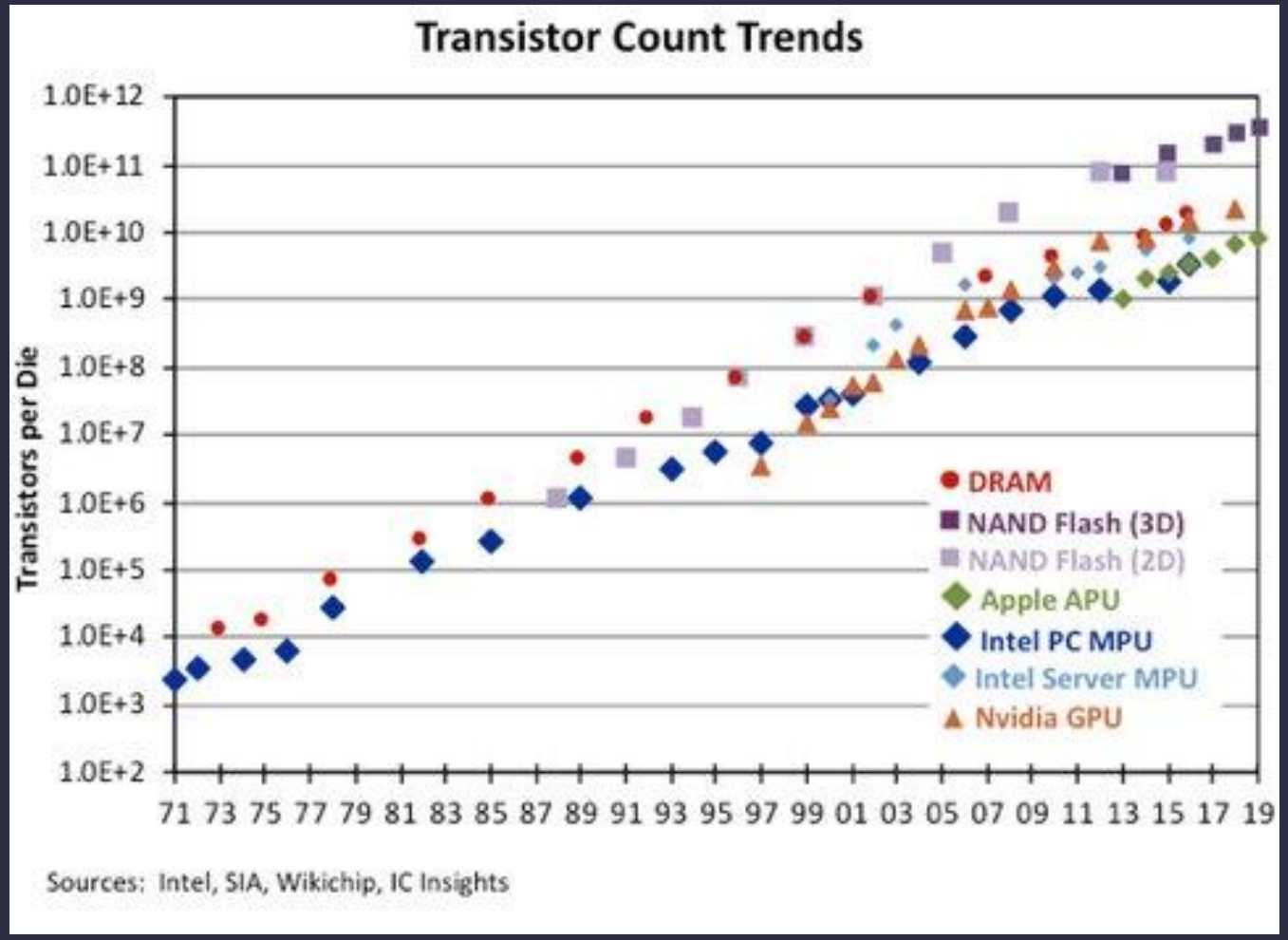
2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
512	256	128	64	32	16	8	4	2	1

5	2	3
10^2	10^1	10^0

		1	0	1
2^4	2^3	2^2	2^1	2^0

Moore 's Rule

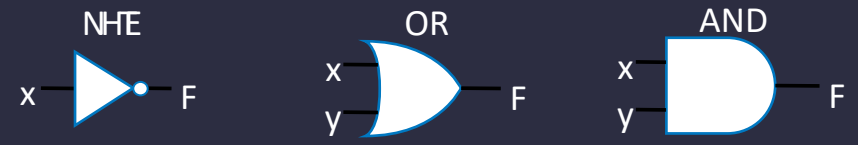
- The rule is that the number of transistorson chips doubles every 18 months .



Boolean Algebra

- Logic Gates are built with

Symbol



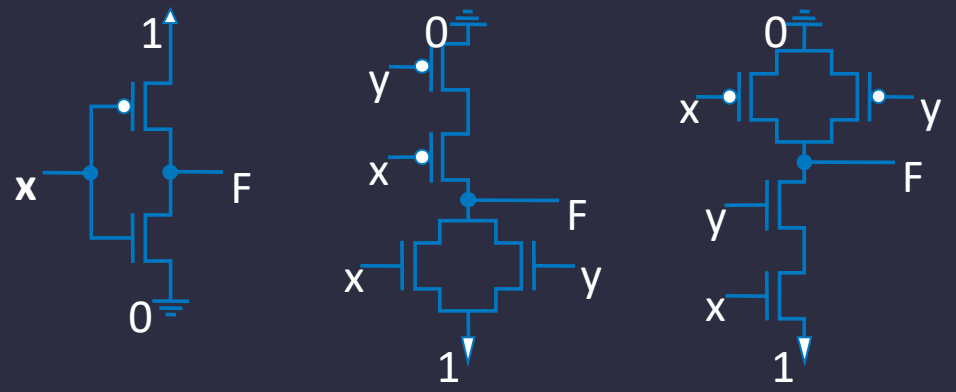
Truth Table

x	F
0	1
1	0

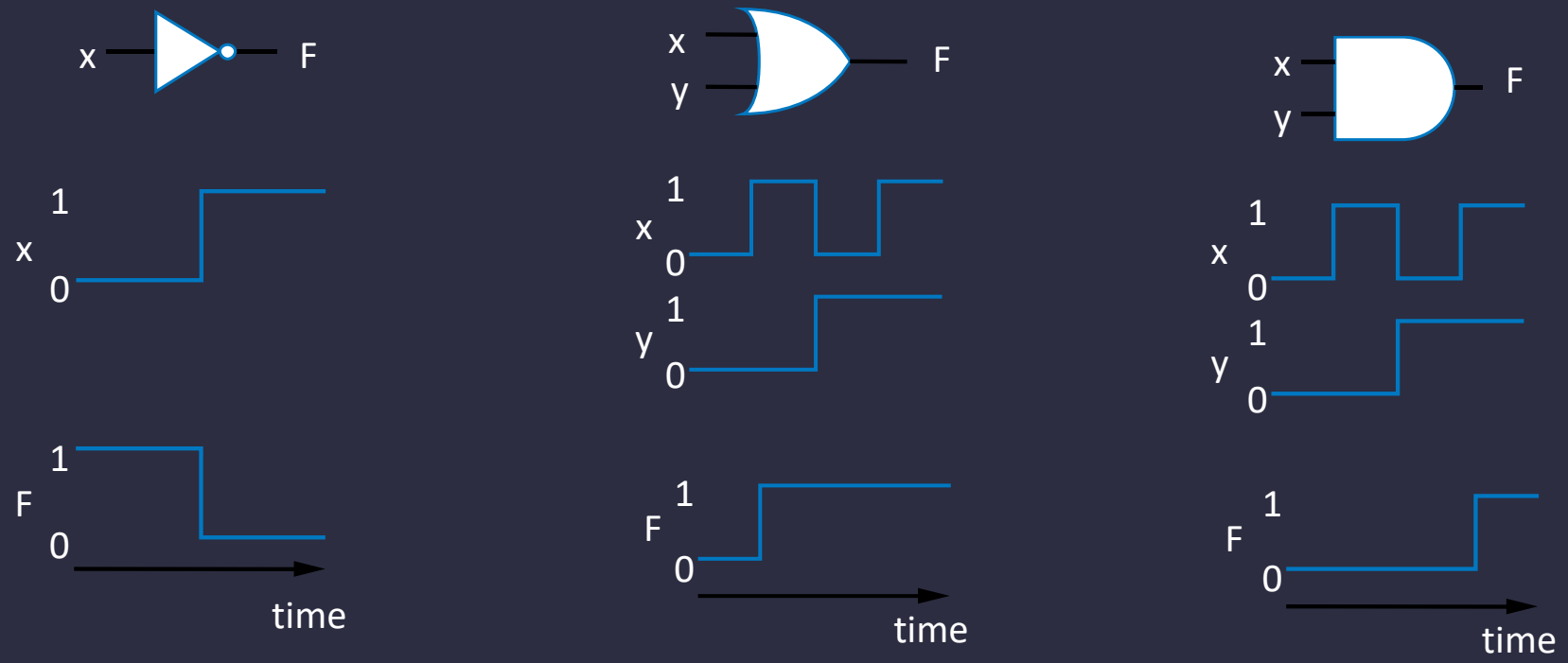
x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

Transistor circuit



NOT/OR/AND Logic Gates Time Diagram



Boolean Algebra Example

- a=1, b=1, c=1, d=0

F = (a **AND b) OR (c AND d)**

Answer : F = (1 AND 1) OR (1 AND 0)
= 1 OR 0 = 1.

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

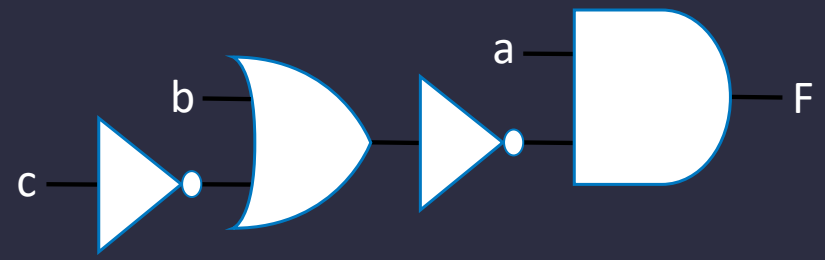
a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

Boolean Algebra Example

- boolean equation

  given below F = a AND NOT(b OR NOT(c))



Boolean Algebra Properties

- Change (Commutative)
  - a + b = b + a
  - a * b = b * a
- Distribution
  - a * (b + c) = a * b + a * c
  - a + (b * c) = (a + b) * (a + c)
- Relational
  - (a + b) + c = a + (b + c)
  - (a * b) * c = a * (b * c)
- Complementary
  - a + a' = 1
  - a * a' = 0

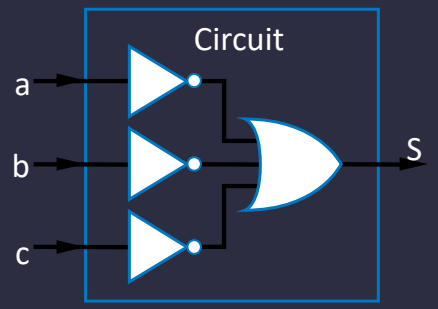
Boolean Algebra Properties

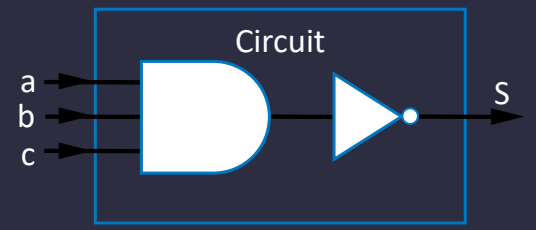
Example

- abc + abc ' = ab
  - Distribution Feature
    - abc + abc ' = ab(c+c ').
  - Association Feature
    - c+c ' has been replaced by 1 : ab(c+c ') = ab(1).
    - ab(1) = ab*1 = ab.

Boolean Algebra

- Operations with constants
  - a + 1 = 1
  - a + 0 = a
  - a * 1 = a
  - a * 0 = 0
- Ineffective Transactions
  - a + a = a
  - a * a = a
- Get the inverse
  - (a')' = a
- DeMorgan's Rule
  - (a + b)' = a'b '
  - (ab)' = a' + b'
- Staring circuit
  - S = a' + b' + c'
- Transformation
  - a'+ b'+ c'
  - ((a'+ b'+ c')')'
  - S = (abc)'





Truth Tables

- F indicates output.
  - 2- Input : 4 lines
  - 3- Input : 8 lines
  - 4- Input : 16 lines

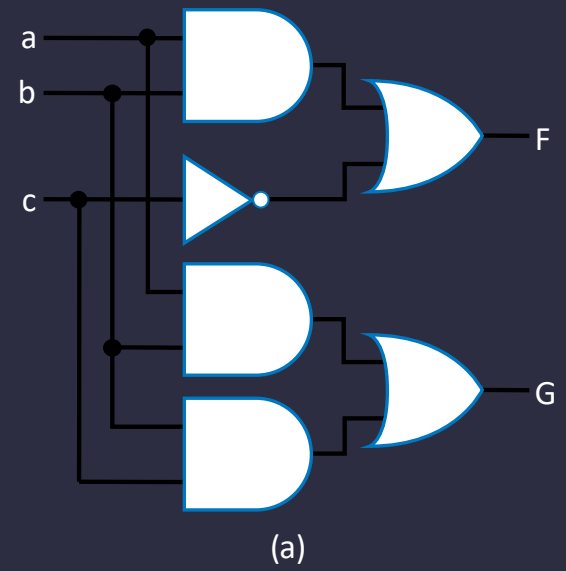
a	b	F
0	0	
0	1	
1	0	
1	1	

a	b	c	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

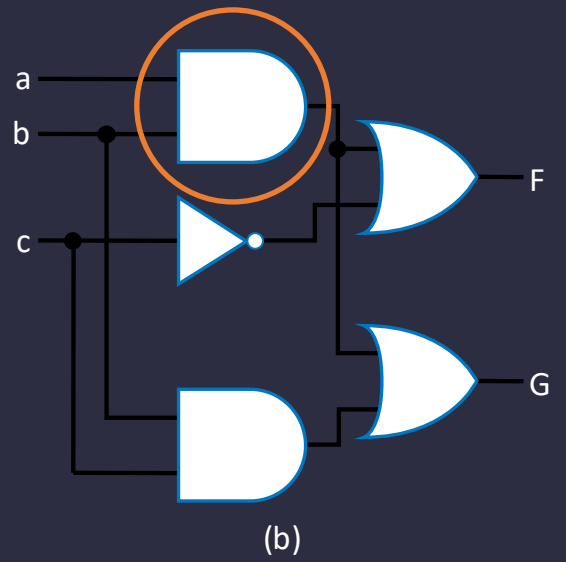
a	b	c	D	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	

Circuit Common Use

- Example : F = ab + c', G = ab + bc



Example 1: Separate Use



Example 2: Common Use