

Digital Design

Week 6: Finite State Machines



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Course

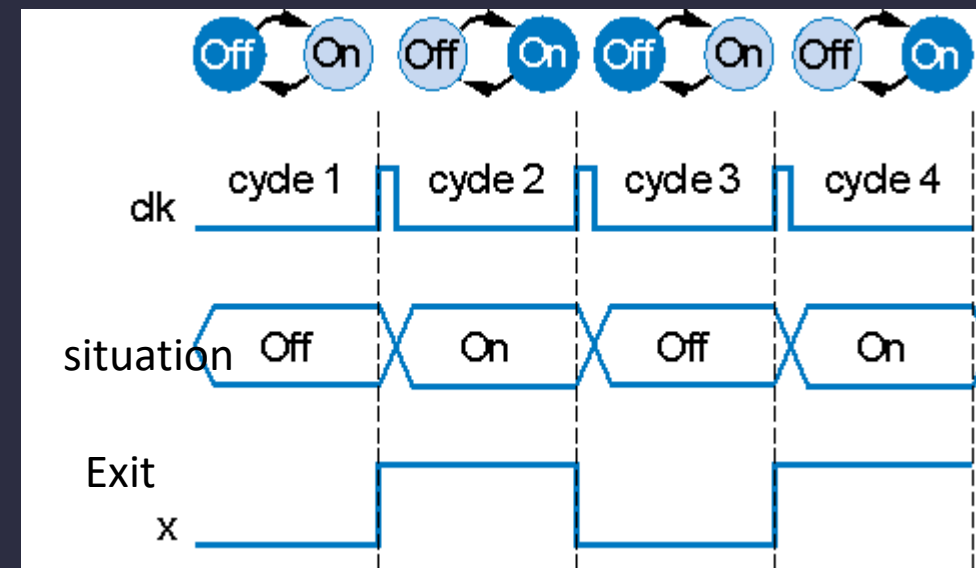
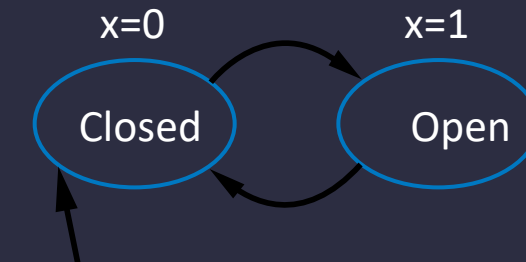
- Finite State Machines

Determining the Behavior of the Sequential Circuit FSM

- Finite-State Machine (FSM)

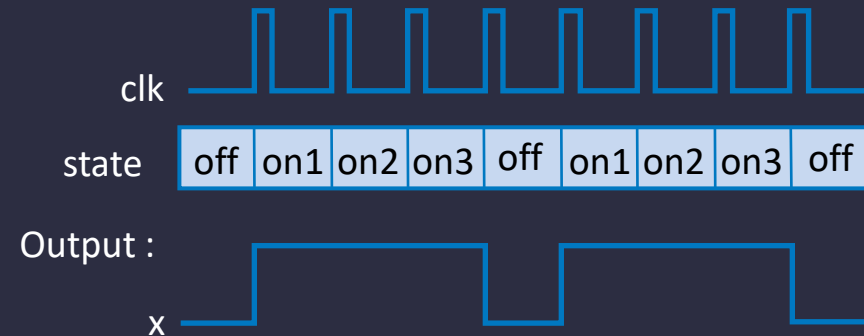
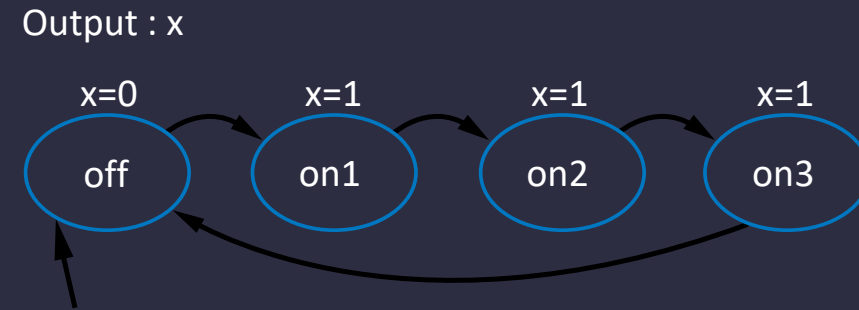
- It is a representation of the behavior of the sequential circuit according to the states.
- Lists states and transitions between states
 - Example : Let there be an output signal named X and each clock change value in cycle
 - Two states : “Off” (x=0), and “On” (x=1)
 - There are transitions from the Off state to the On state and from the On state to the Off state.
 - The initial state is indicated by an arrow.

Output : x



FSM Example : 0,1,1,1, repetition

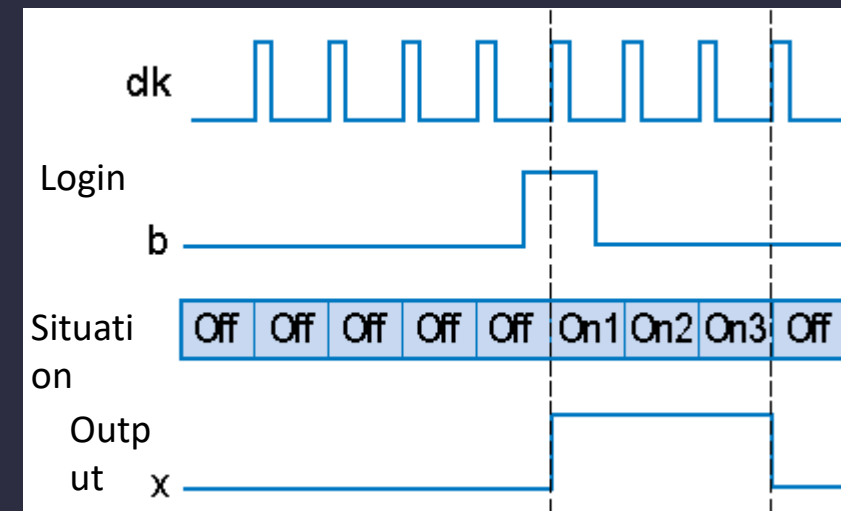
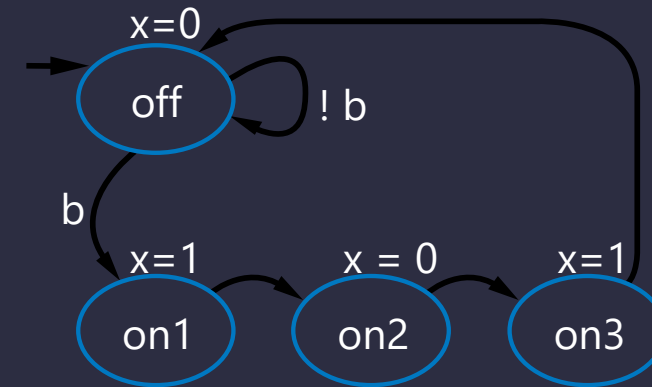
- In order 0, 1, 1, 1, 0, 1, 1, 1, ... A circuit that gives its outputs will be designed.
 - Each value is a clock It appears in the cycle .
- Can be designed with FSM
 - There are 4 states



FSM Example

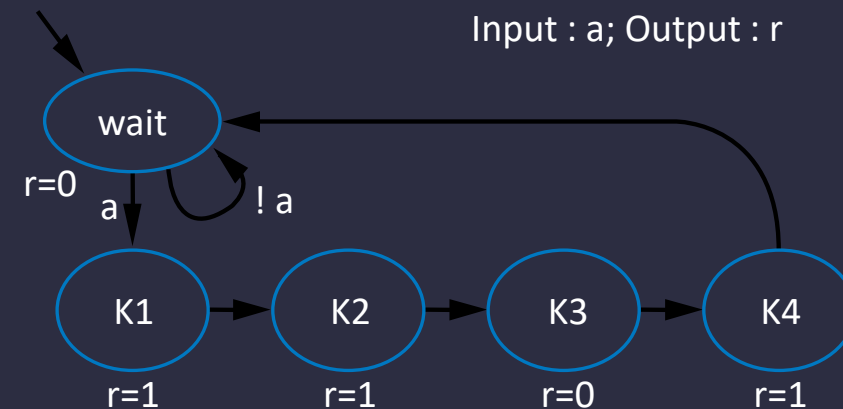
take a one-bit input named b ,
Output pattern 101 and waits for
input b again.

Inputs : b ; Outputs : x



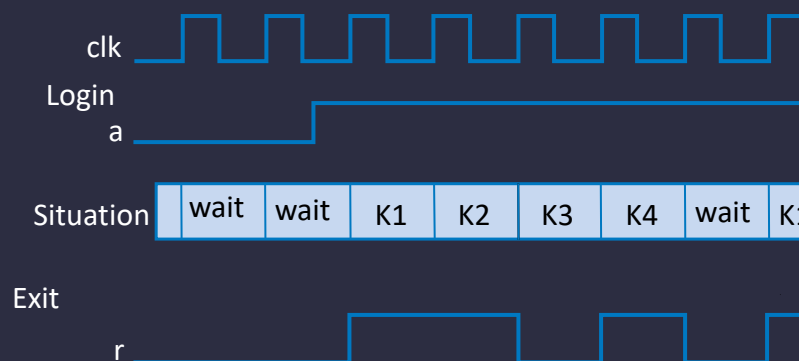
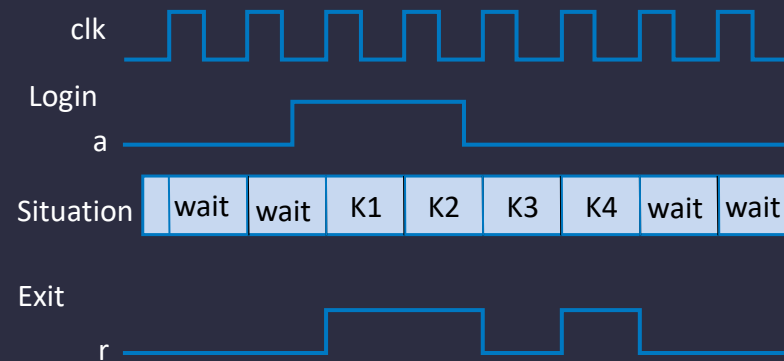
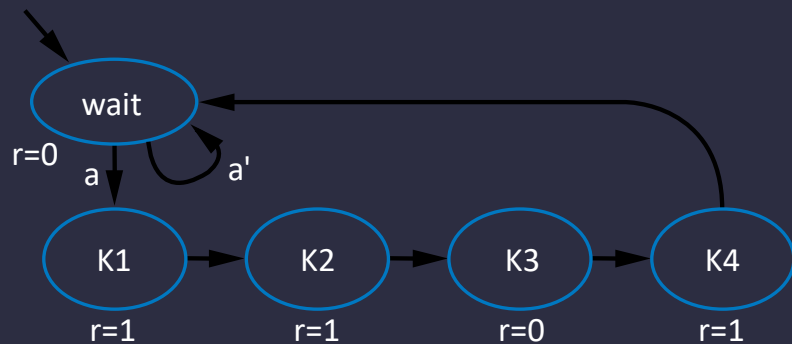
FSM Example: Car Key

- Car keys contain a chip that holds the key's identity.
 - When the key is inserted in the car, it asks the key for identification.
 - Sends key credential, if not correct the tool won't start
- FSM
 - Wait for request ($a=1$)
 - Send ID (For example , 1101)



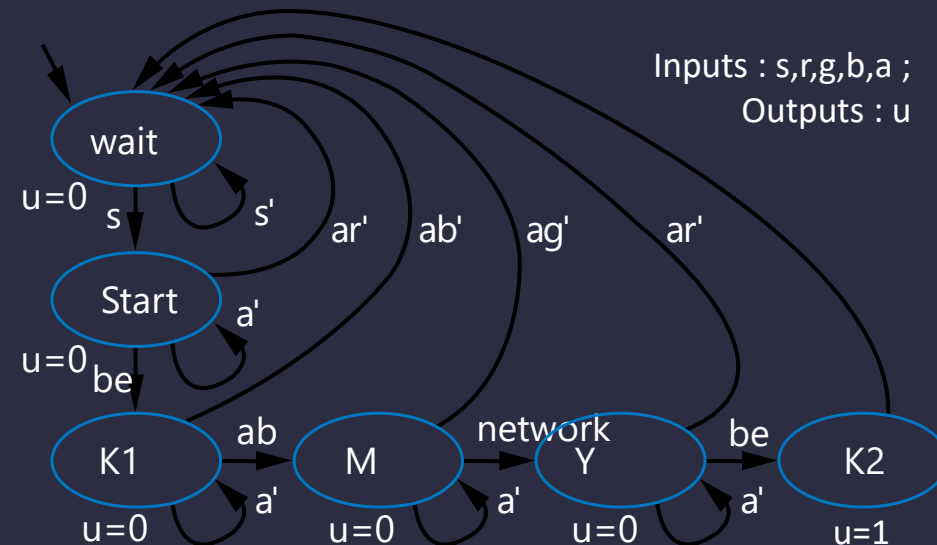
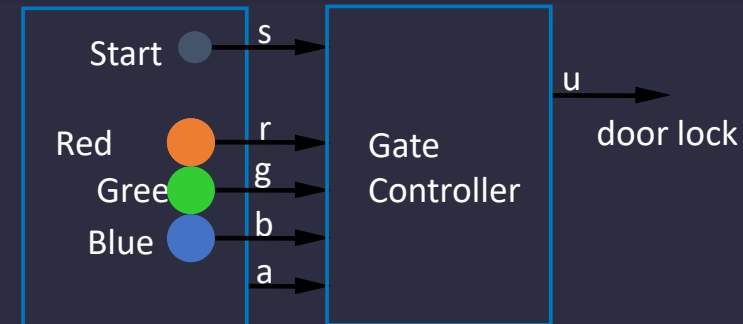
FSM Example: Car Key

- FSM Timings

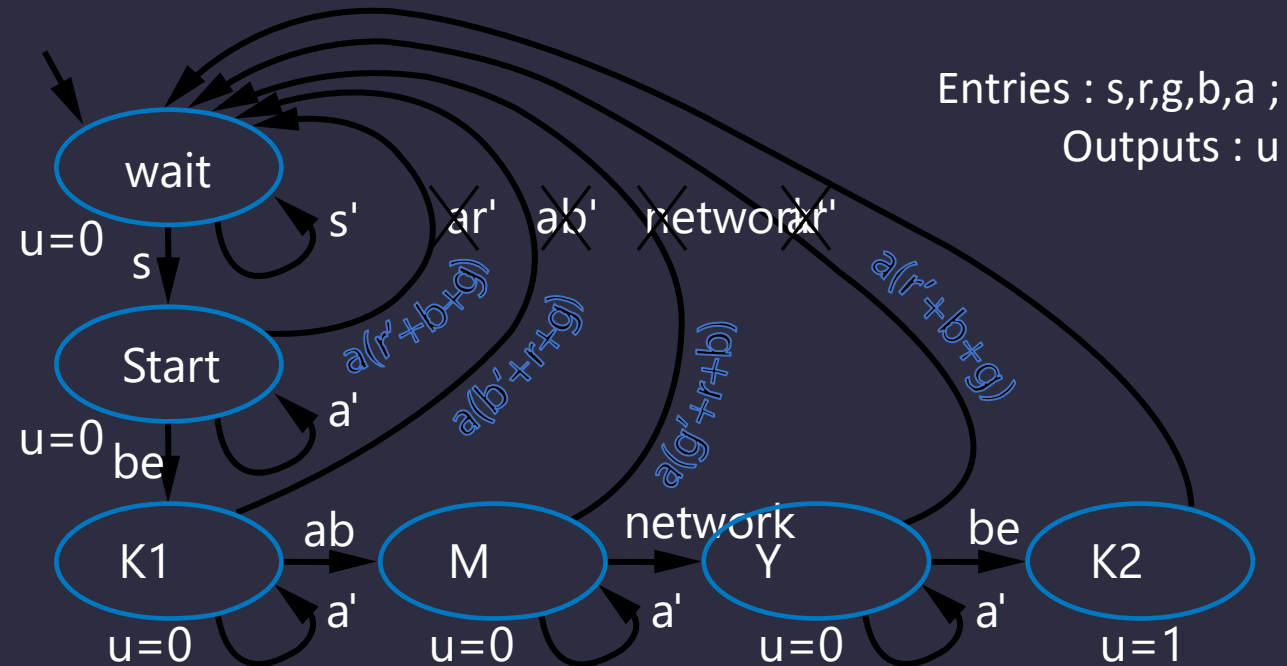


FSM Example: Code Detector

- A door is only opened when the following button sequence is pressed ($u=1$)
 - start , red , blue , green , red
- s, r, g, b
 - If input a , buttons of other color
- FSM
 - While in wait state, start when start
 - When start is pressed
 - If it's red, go to K1
 - Then, if it's blue, M
 - Then, if green, Y
 - Then, if it's red, K2
 - In this case, the signal to open the door is generated ($u=1$)
 - Any wrong input will revert to wait state



FSM Example: Code Detector, Improvements

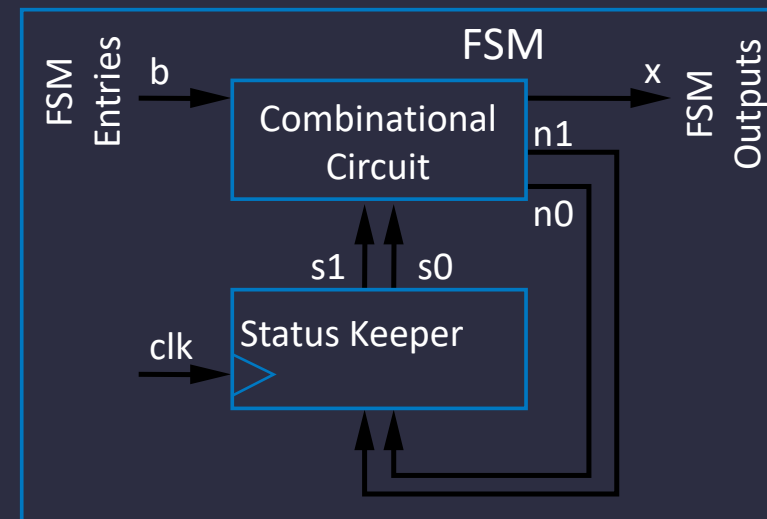
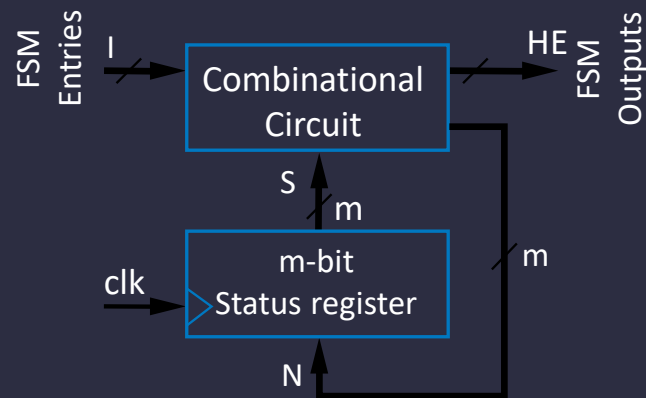
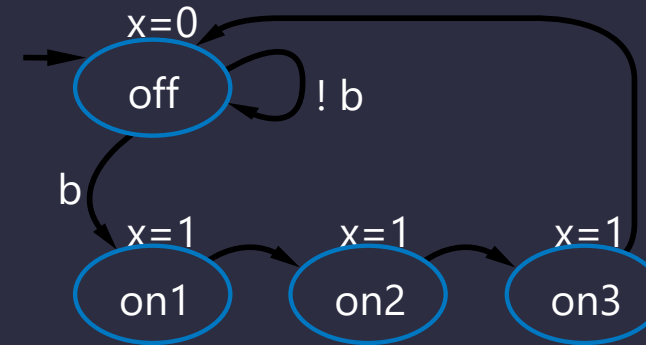


In the new state machine, pressing the wrong button returns to the "Wait" state.

FSM Architecture

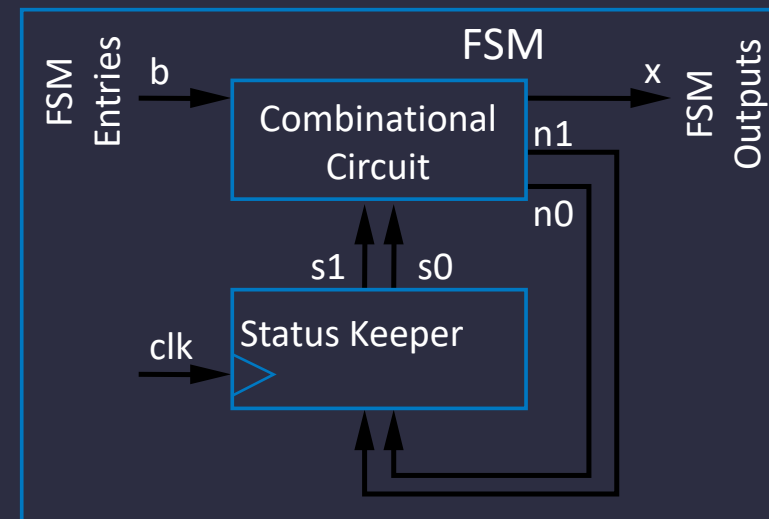
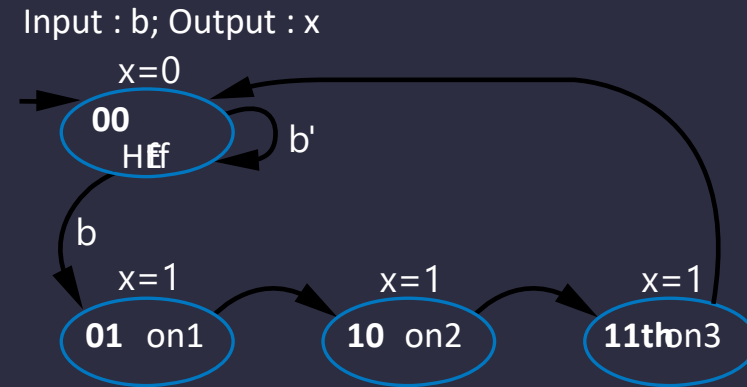
- How is FSM implemented as a sequential circuit ?
 - Status register - to keep the current state
 - Combinational circuit – to calculate output and next state

Input : b; Output : x



FSM Design Example

- Step 1: Analyze the requirements
- Step 2: Extract architecture
 - 2-bit state register (for 4 states)
 - Input b, output x
 - Next status signals n1, n0
- Step 3: Encode the states

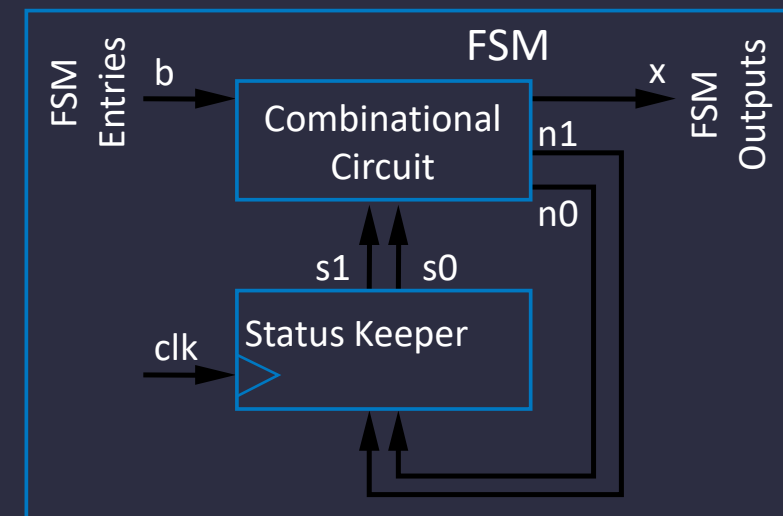
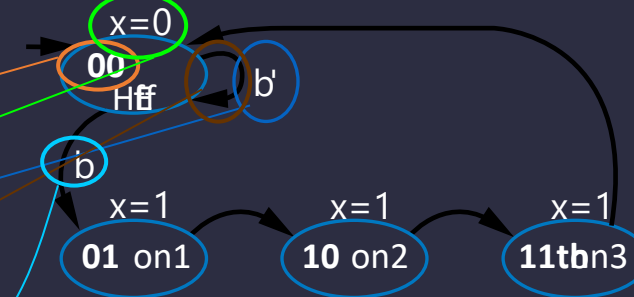


FSM Design Example

- Step 4: Create a Truth Table

	Entries			Outputs		
	s1	s0	b	x	n1	n0
<i>Off</i>	0	0	0	0	0	0
	0	0	1	0	0	1
<i>On1</i>	0	1	0	1	1	0
	0	1	1	1	1	0
<i>On2</i>	1	0	0	1	1	1
	1	0	1	1	1	1
<i>On3</i>	1	1	0	1	0	0
	1	1	1	1	0	0

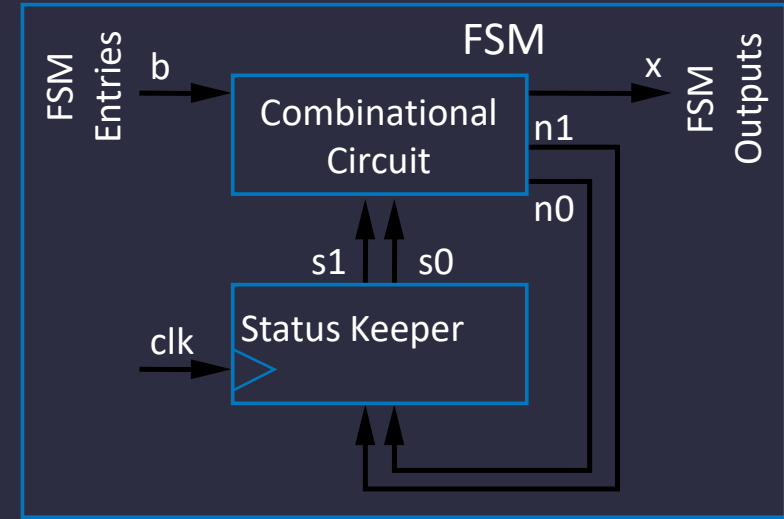
Inputs: b; Outputs: x



FSM Design Example

- Step 5: Combinational Circuit Implementation

	Entries			Outputs		
	s1	s0	b	x	n1	n0
<i>Off</i>	0	0	0	0	0	0
	0	0	1	0	0	1
<i>On1</i>	0	1	0	1	1	0
	0	1	1	1	1	0
<i>On2</i>	1	0	0	1	1	1
	1	0	1	1	1	1
<i>On3</i>	1	1	0	1	0	0
	1	1	1	1	0	0



$$x = s1 \mid s0$$

$$n1 = s1's0b' \mid s1's0b \mid s1s0'b' \mid s1s0'b$$

$$n1 = s1's0 \mid s1s0'$$

$$n0 = s1's0'b \mid s1s0'b' \mid s1s0'b$$

$$n0 = s1's0'b \mid s1s0'$$

FSM Design Example

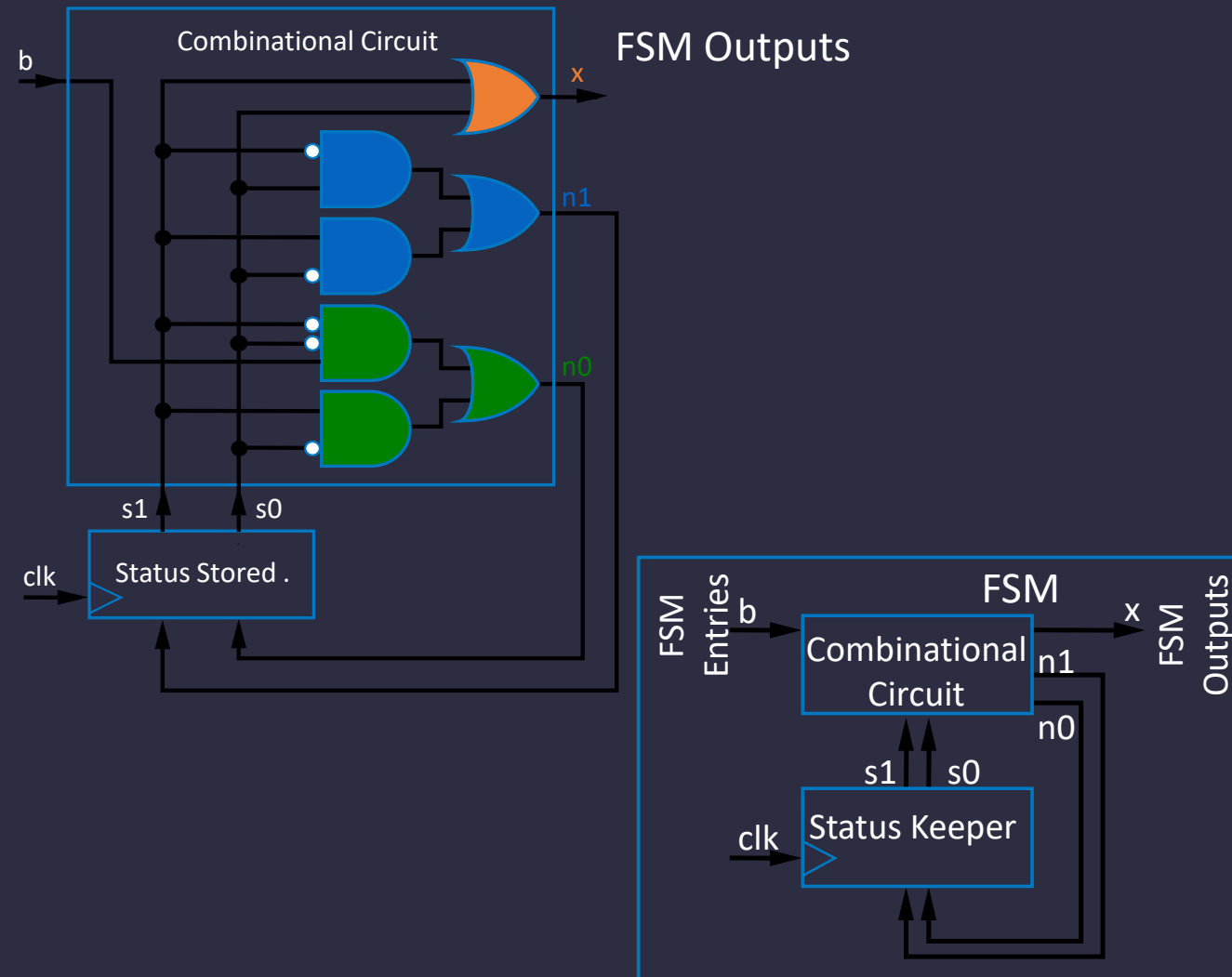
- Step 5: Build the combinational circuit

	Entries			Output		
	s1	s0	b	x	n1	n0
<i>Off</i>	0	0	0	0	0	0
	0	0	1	0	0	1
<i>On1</i>	0	1	0	1	1	0
	0	1	1	1	1	0
<i>On2</i>	1	0	0	1	1	1
	1	0	1	1	1	1
<i>On3</i>	1	1	0	1	0	0
	1	1	1	1	0	0

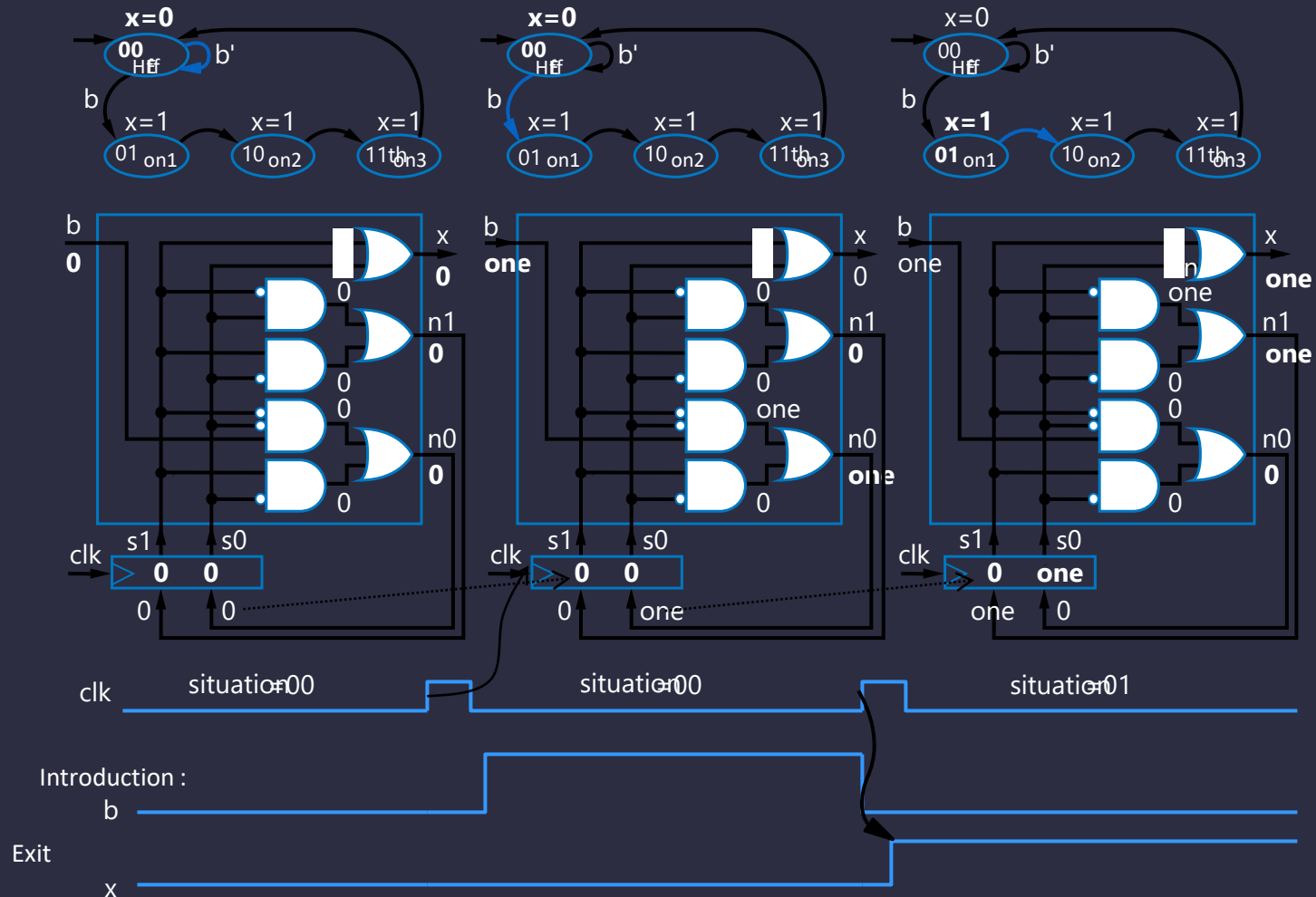
$$x = s1 \mid s0$$

$$n1 = s1's0 \mid s1s0'$$

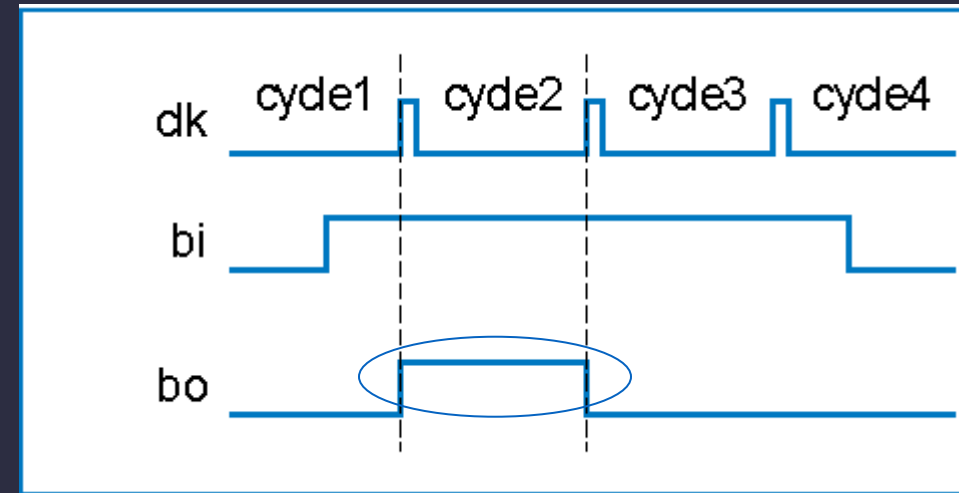
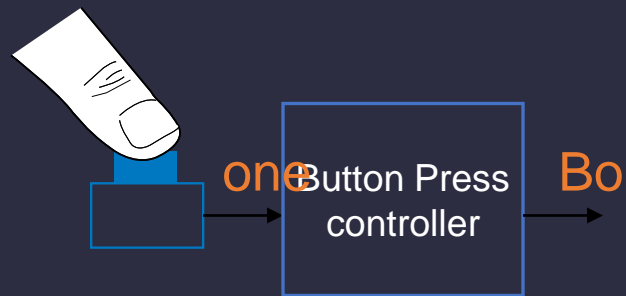
$$n0 = s1's0'b \mid s1s0'$$



Internal Structure of



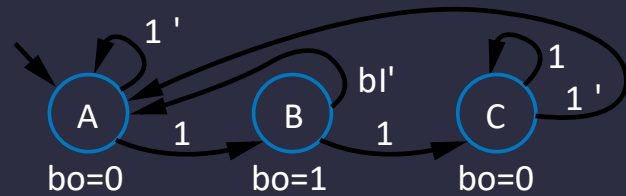
Button Filter Module Example



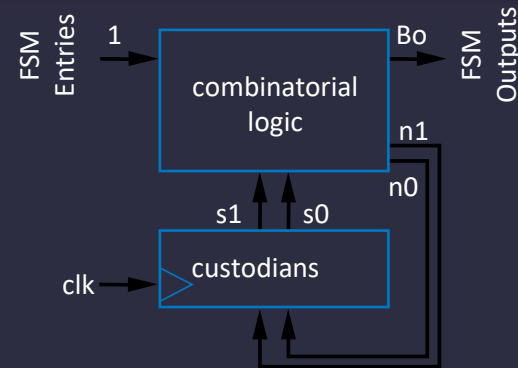
- cycle when the button is pressed A design that produces a pulse is desired.

Button Filter Module Example

FSM Inputs : bi; FSM outputs : bo



Step 1: FSM



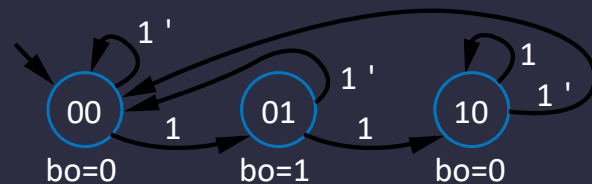
Step 2: Build architecture

$$n1 = s1's0bi \mid s1s0bi$$

$$n0 = s1's0'bi$$

$$bo = s1's0bi \mid s1's0bi = s1s0$$

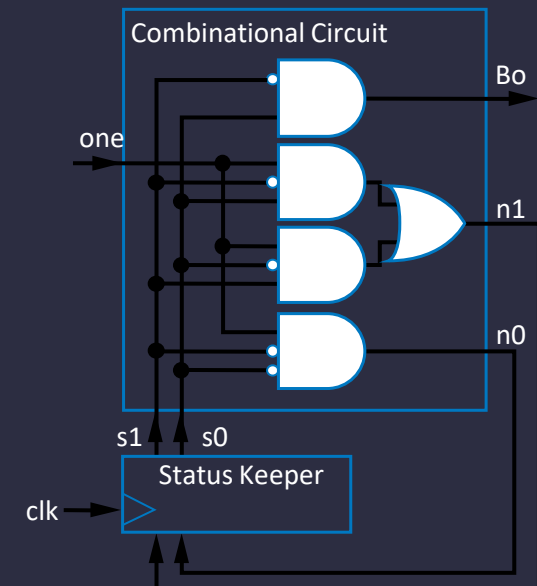
FSM input : bi; FSM output : bo



Step 3: State coding

Entries			Outputs		
s1	s0	bi	n1	n0	bo
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	0	0

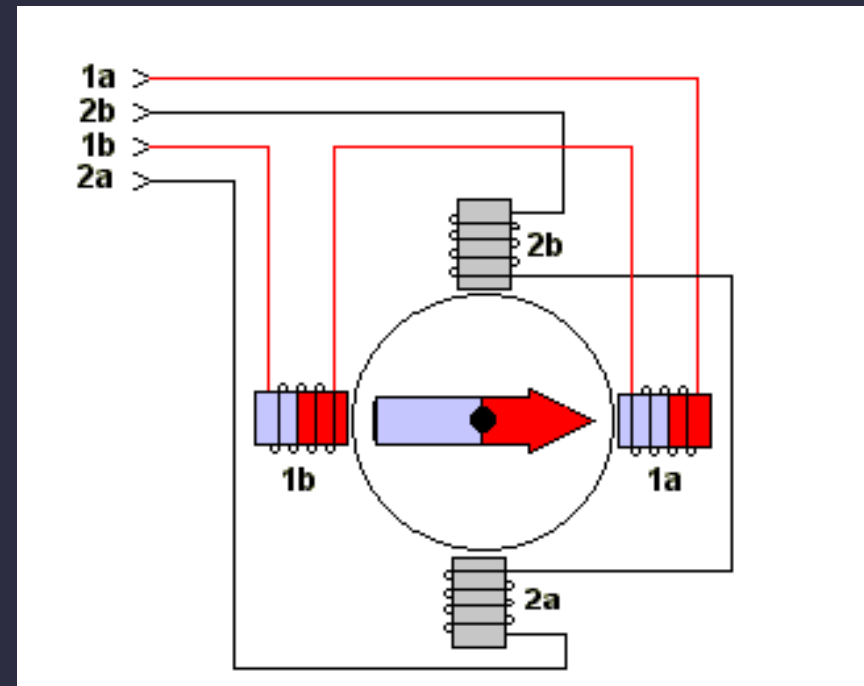
Step 4: Status Chart



Step 5: Build Circuit

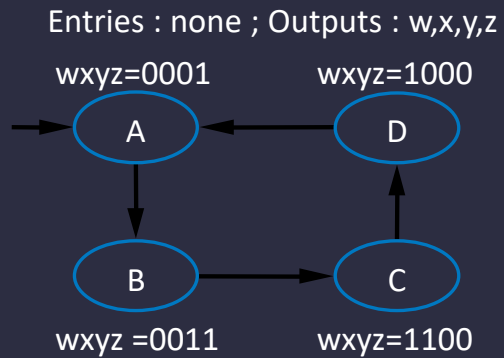
FSM Example

- Design an FSM that outputs 0001, 0011, 1100, 1000 sequentially.
 - For example, for stepper motor control



FSM Example

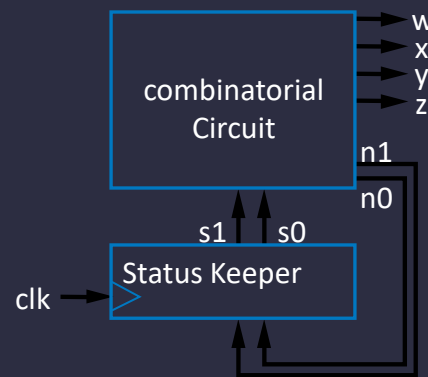
- Design an FSM that outputs 0001, 0011, 1100, 1000 sequentially.
 - For example, for stepper motor control



Step 1: Creating the FSM

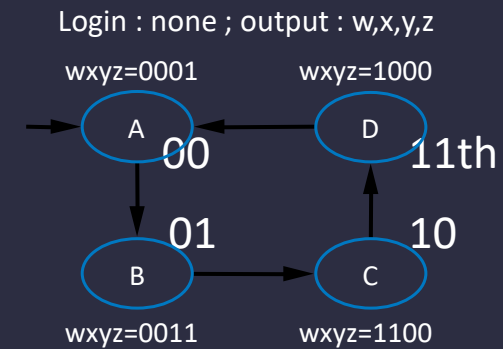
Entries	Outputs							
	s1	s0	w	x	y	z	n1	n0
A	0	0	0	0	0	1	0	1
B	0	1	0	0	1	1	1	0
C	1	0	1	1	0	0	1	1
D	1	1	1	0	0	0	0	0

Step 4: Create a Truth Table

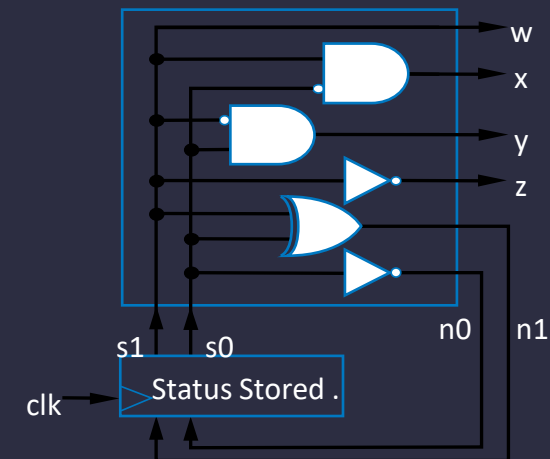


Step 2: Creating Architecture

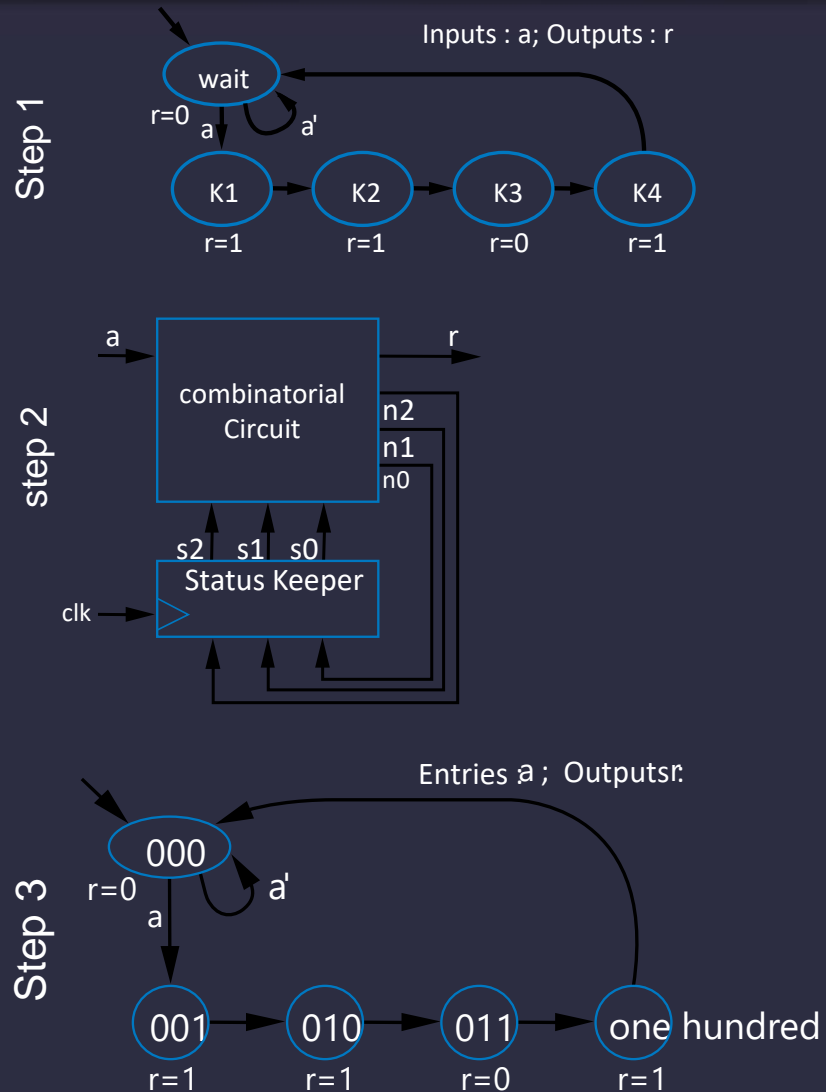
$$\begin{aligned}
 w &= s1 \\
 x &= s1s0' \\
 y &= s1's0 \\
 z &= s1' \\
 n1 &= s1 \text{ x or } s0 \\
 n0 &= s0'
 \end{aligned}$$



Step 3: State Coding



FSM Example, Car Key

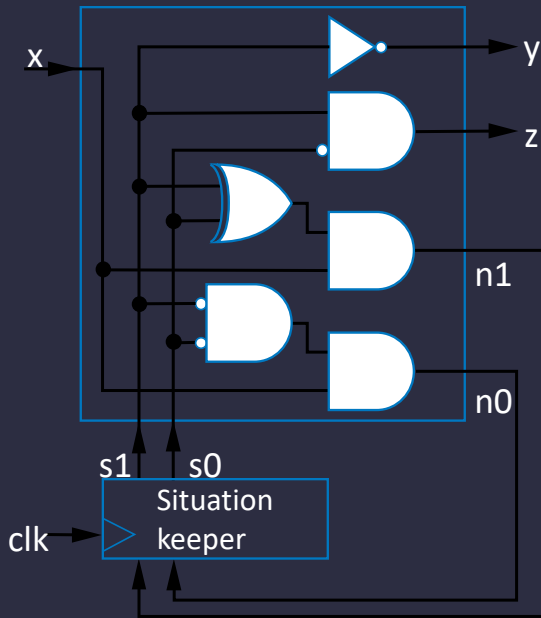


	Entries				Outputs			
	s2	s1	s0	a	r	n2	n1	n0
wait	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	1
K1	0	0	1	0	1	0	1	0
	0	0	1	1	1	0	1	0
K2	0	1	0	0	1	0	1	1
	0	1	0	1	1	0	1	1
K3	0	1	1	0	0	1	0	0
	0	1	1	1	0	1	0	0
K4	1	0	0	0	1	0	0	0
	1	0	0	1	1	0	0	0
-	1	0	1	0	0	0	0	0
	1	0	1	1	0	0	0	0
	1	1	0	0	0	0	0	0
	1	1	0	1	0	0	0	0
	1	1	1	0	0	0	0	0
	1	1	1	1	0	0	0	0

Step 4

Reverting to

What does this circuit do?



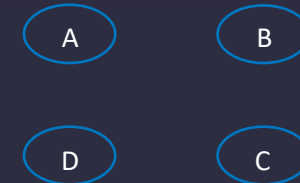
$$y = s1'$$

$$z = s1s0'$$

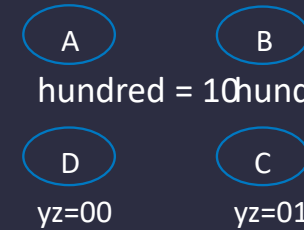
$$n1 = (s1x \text{ or } s0)x$$

$$n0 = (s1's0')x$$

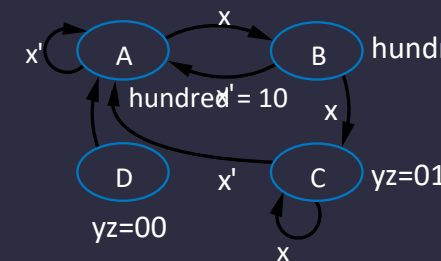
	Entries			Outputs			
	s1	s0	x	n1	n0	y	z
A	0	0	0	0	0	1	0
	0	0	1	0	1	1	0
B	0	1	0	0	0	1	0
	0	1	1	1	0	1	0
C	1	0	0	0	0	0	1
	1	0	1	1	0	0	1
D	1	1	0	0	0	0	0
	1	1	1	0	0	0	0



situations



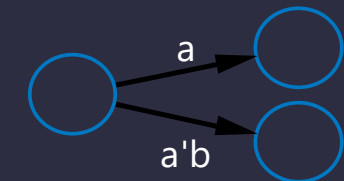
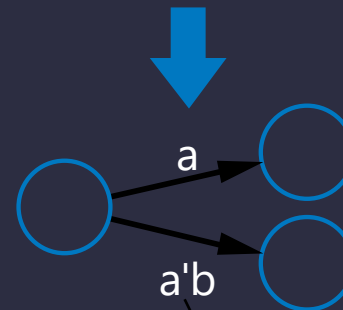
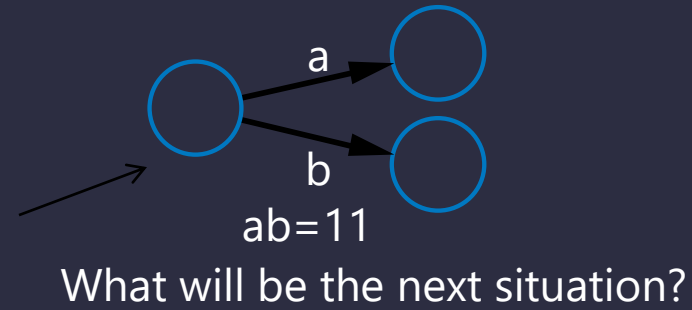
States and exits



States, exits and transition

FSM Design Errors

- In conditions in transitions between states, only one transition condition must be true at a time*



ab=00?
What will be the next situation?

