Electronic Circuits

Week 5: Parallel Circuit

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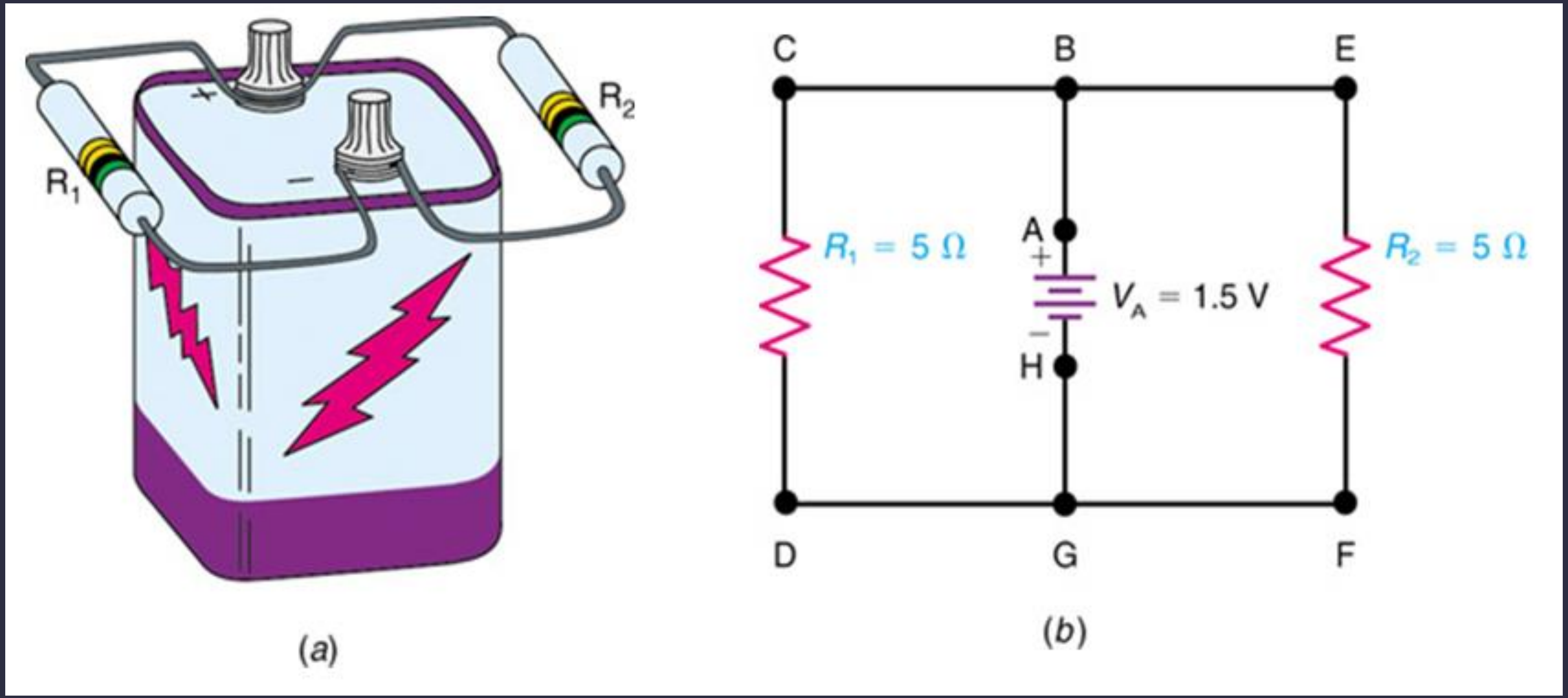
The Applied Voltage V_A Is the Same Across Parallel Branches

Characteristics of a Parallel Circuit.

- Voltage is the same across each branch in a parallel circuit.
- The total current is equal to the sum of the individual branch currents.
- The equivalent resistance (R_{EQ}) is less than the smallest branch resistance. The term equivalent resistance refers to a single resistance that would draw the same amount of current as all of the parallel connected branches.
- Total power is equal to the sum of the power dissipated by each branch resistance.

The Applied Voltage V_A Is the Same Across Parallel Branches

Example of a parallel circuit with two resistors.
(a) Wiring diagram.
(b) Schematic diagram.



Each Branch *I* Equals V_A / R

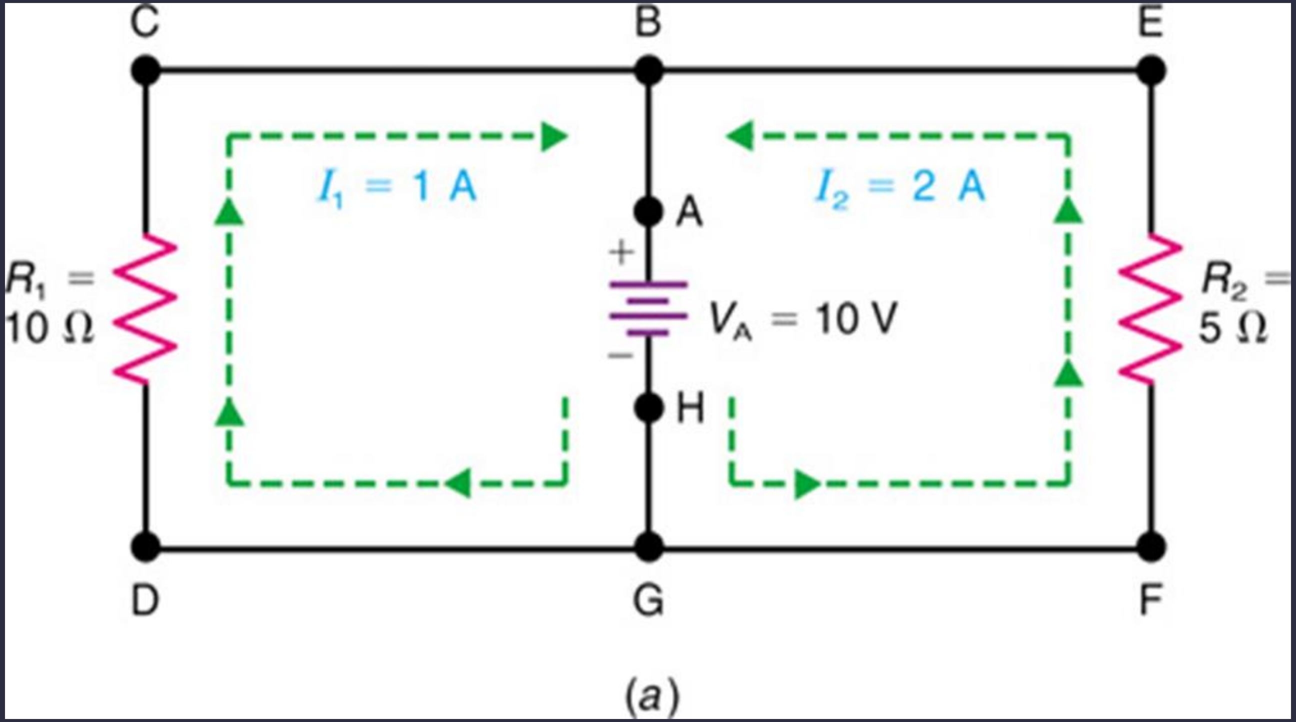
- The current in a parallel circuit equals the voltage applied across the circuit divided by the resistance between the two points where the voltage is applied.
- Each path for current in a parallel circuit is called a **branch**.

Each branch current equals $\frac{V}{R}$ where *V* is the same across all branches.

Each Branch I Equals V_A / R

Parallel circuit.

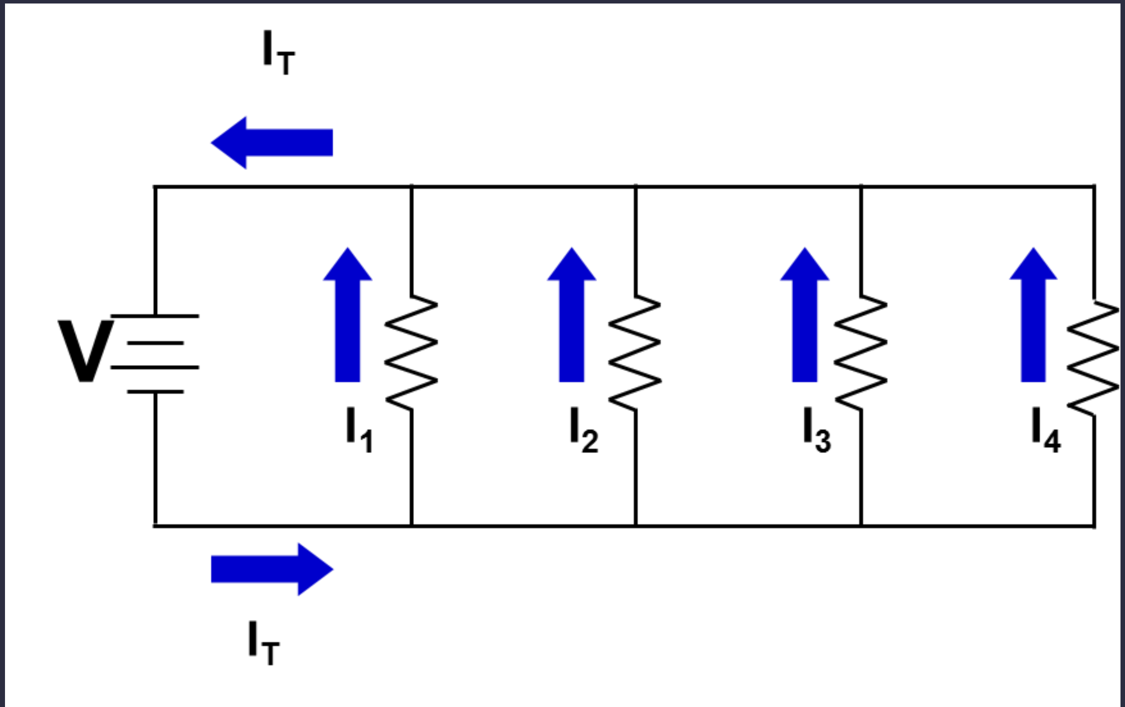
(a) the current in each parallel branch equals the applied voltage V_A divided by each branch resistance R.



Kirchhoff's Current Law (KCL)

- The pair of leads connecting all the branches to the voltage source terminals is the **main line**.
- All the current in the circuit must come from one side of the voltage source and return to the opposite side for a complete path.
- The amount of current in the main line is equal to the total of the branch currents.

Kirchhoff’s Current Law (KCL)



$$I_T = I_1 + I_2 + I_3 + I_4$$

- The total current I_T in the main line is equal to the sum of the branch currents.
- This is known as Kirchhoff’s current law (KCL).
- It applies to any number of parallel branches, whether the resistances in those branches are equal or not.

Resistance in Parallel

- The combined equivalent resistance of a parallel circuit may be found by dividing the common voltage across all resistances by the total current of all the branches.

$$R_{EQ} = \frac{V_A}{I_T}$$

Resistance in Parallel

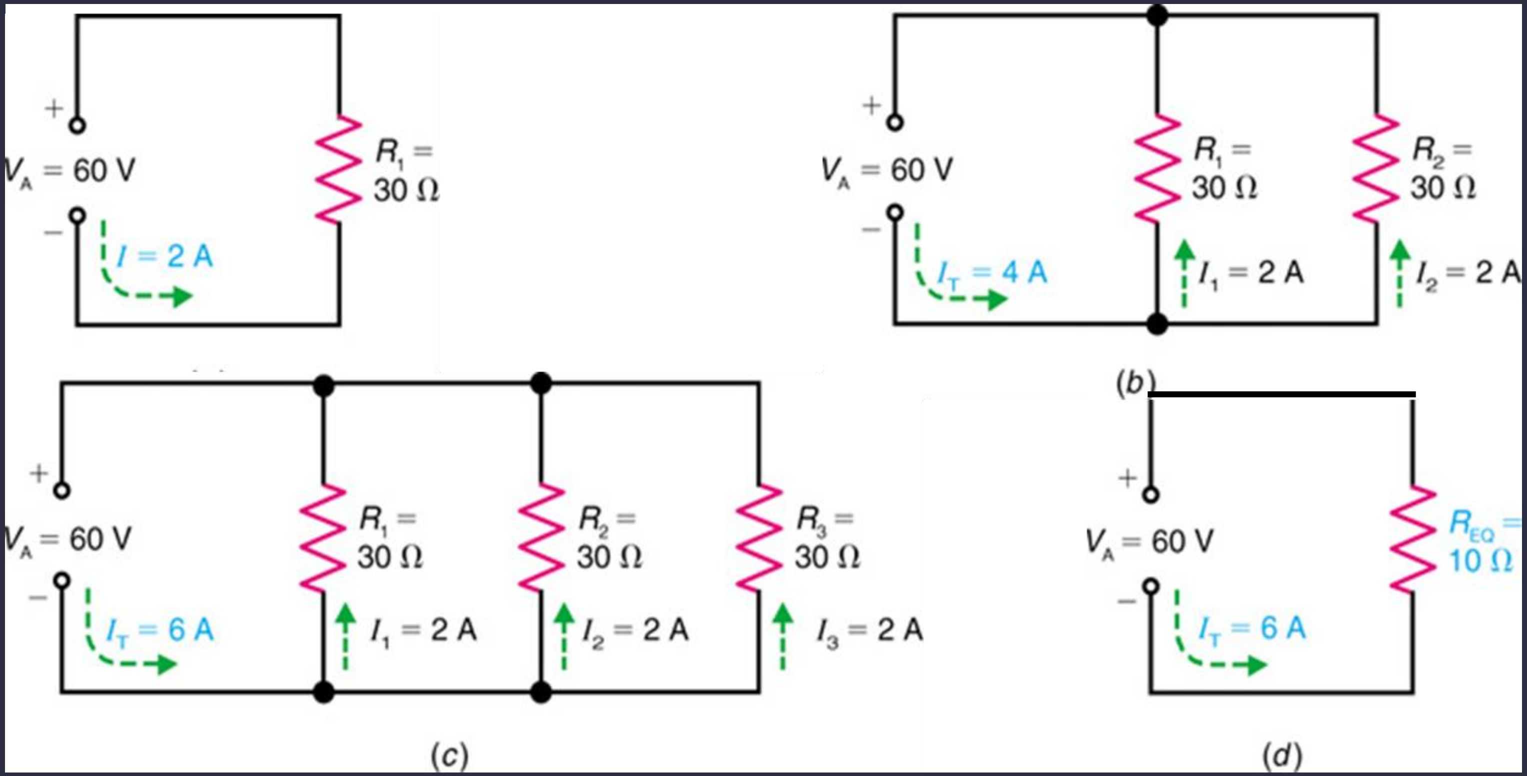
- The equivalent resistance of a parallel circuit must be less than the smallest branch resistance.

- Adding more branches to a parallel circuit reduces the equivalent resistance because more current is drawn from the same voltage source.

Resistance in Parallel

How adding parallel branches of resistors increases I_T but decreases R_{EQ}.

(a) One resistor. (b) Two branches. (c) Three branches. (d) Equivalent circuit of the three branches in (c).



Resistance in Parallel

Total Current and Reciprocal Resistance Formulas.

- In a parallel circuit, the total current equals the sum of the individual branch currents:

$$I_T = I_1 + I_2 + I_3 + \ldots + \text{etc.}$$

- Total current is also equal to total voltage divided by equivalent resistance:

$$I_T = \frac{V_T}{R_{EQ}}$$

Resistance in Parallel

Total Current and Reciprocal Resistance Formulas.

- The equivalent resistance of a parallel circuit equals the reciprocal of the sum of the reciprocals:

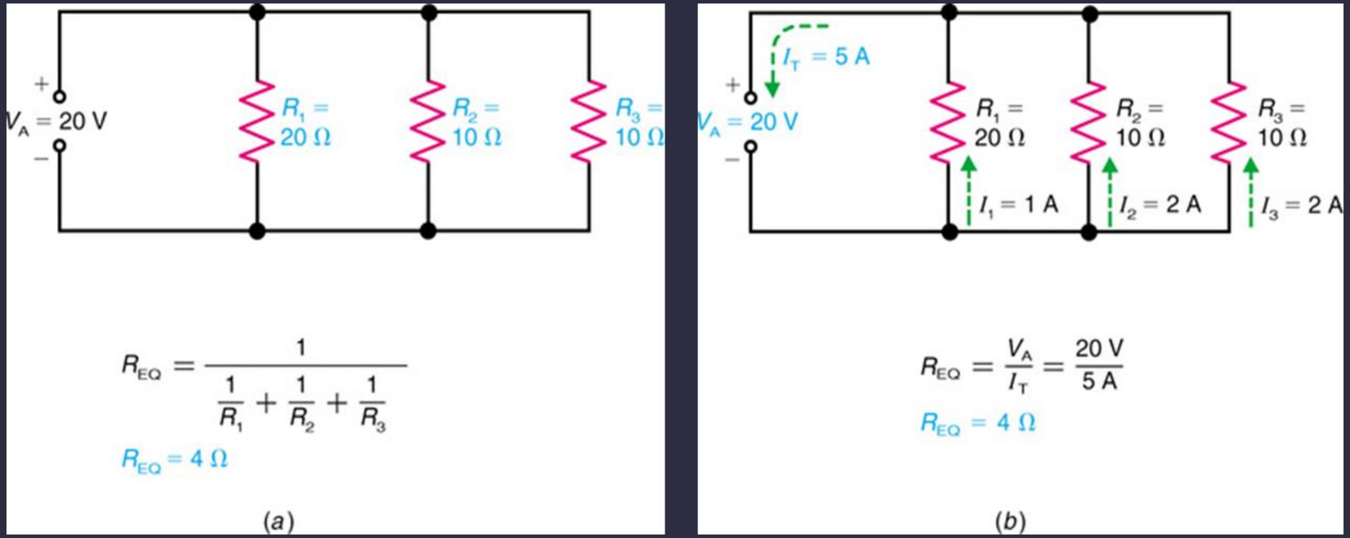
$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \text{ etc.}}$$

Equivalent resistance also equals the applied voltage divided by the total current:

$$R_{EQ} = \frac{V_A}{I_T}$$

Resistance in Parallel

- Determining the Equivalent Resistance.



Two methods of combining parallel resistances to find R_{EQ}. (a) Using the reciprocal

resistance formula to calculate R_{EQ} as 4Ω. (b) Using the total line current method with an assumed line voltage of 20 V gives the same 4Ω for R_{EQ}.

Resistance in Parallel

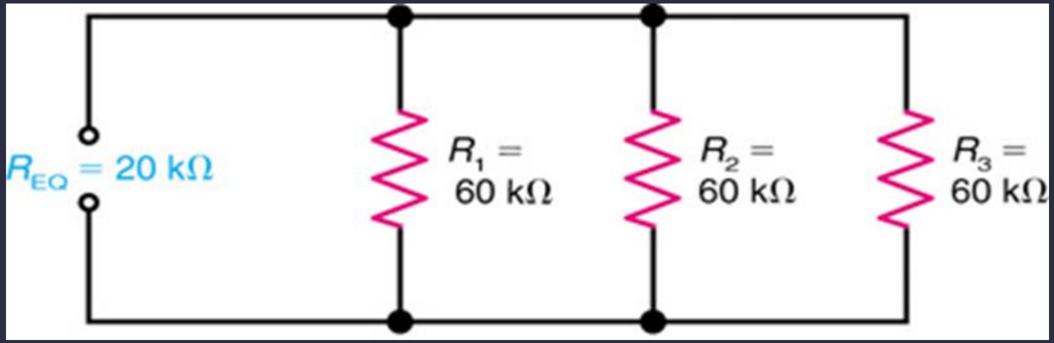
Special Case: Equal Value Resistors.

- If *R* is equal in all branches, divide one resistor's value by the number of resistors (N).

$$R_{EQ} = \frac{R}{N}$$

$$R_{EQ} = \frac{60\ K\Omega}{3\ \text{resistors}}$$

$$R_{EQ} = 20\ \text{K}\Omega$$



$$R_{\text{EQ}} = \frac{\text{value of one resistance}}{\text{number of resistances}} = \frac{60\ \text{k}\Omega}{3}$$

For the special case of all branches having the same resistance, just divide R by the number of branches (N) to find R_{EQ}. Here,

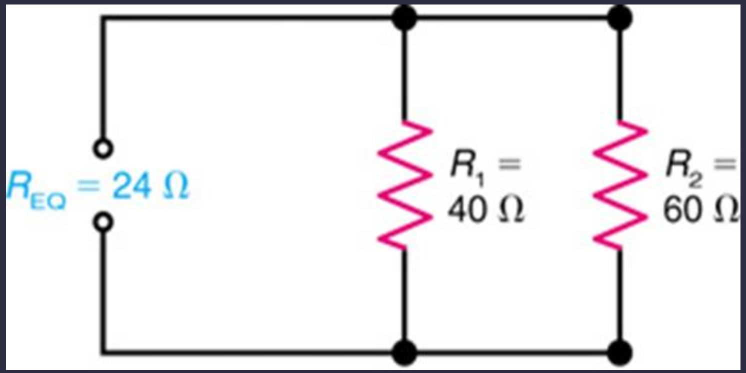
$$\text{R}_{\text{EQ}} = \frac{60\ \text{k}\Omega}{3 = 20\ \text{k}\Omega}$$

Resistance in Parallel

Special Case: Two Unequal Resistors.

- When there are only two branches in a parallel circuit and their resistances are unequal, use the formula:

$$R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2}$$



$$R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{2400}{100}$$

For the special case of only two branch resistances, of any values, R_{EQ} equals

their product divided by the sum. Here, $R_{EQ} = \frac{2400}{100 = 24\ \Omega}$

Resistance in Parallel

- To find an unknown branch resistance, rewrite the formula as follows to solve for the unknown value.

$$R_X = \frac{R \times R_{EQ}}{R - R_{EQ}}$$

- These formulas may be used to simplify complex circuits.

Total Power in Parallel Circuits

- Total power is equal to the sum of the power dissipated by the individual resistances of the parallel branches:

$$P_T = P_1 + P_2 + P_3 + \ldots + \text{etc.}$$

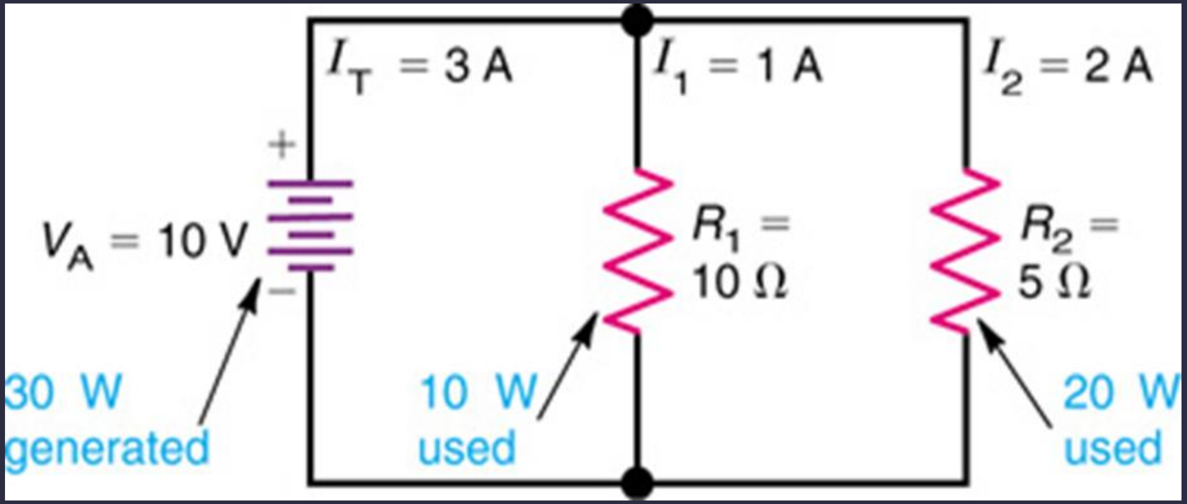
- Total power is equal to voltage times total current:

$$P_T = V_T I_T$$

Total Power in Parallel Circuits

- Determining Power.

$$P_1 = \frac{10^2}{10\ \Omega} = 10\ \text{W}$$
$$P_2 = \frac{10^2}{5\ \Omega} = 20\ \text{W}$$
$$P_T = 10 + 20 = 30\ \text{W}$$



Check: $P_T = V_T \times I_T = 10\ \text{V} \times 3\ \text{A} = 30\ \text{W}$

The sum of the power values P_1 and P_2 used in each branch equals the total power P_T produced by the source.

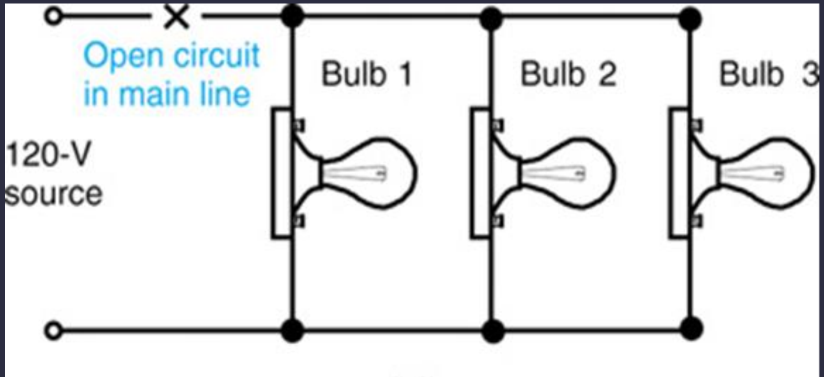
Troubleshooting: Opens and Shorts in Parallel Circuits

Opens in Parallel Circuits.

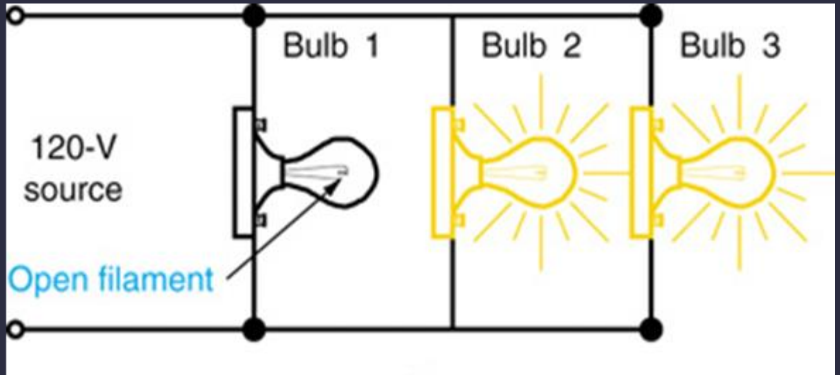
- An open circuit in one branch results in no current through that branch.
- However, an open circuit in one branch has no effect on the other branches. This is because the other branches are still connected to the voltage source.
- An open in the main line prevents current from reaching any branch, so all branches are affected.

Troubleshooting: Opens and Shorts in Parallel Circuits

- Opens in Parallel Circuits.



(a)



(b)

In Figure (b) bulbs 2 and 3 still light. However, the total current is smaller.

In (a) no bulbs light.

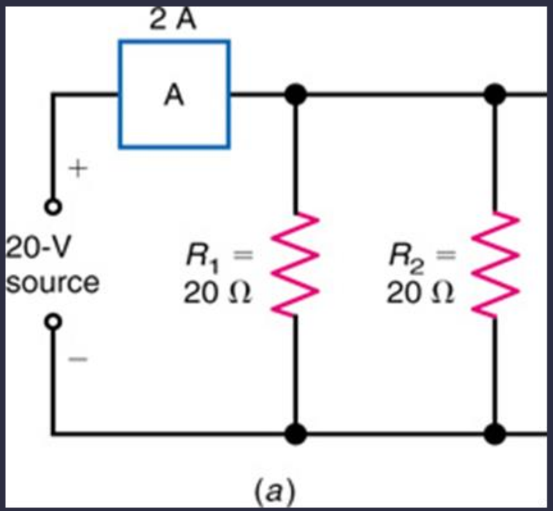
Troubleshooting: Opens and Shorts in Parallel Circuits

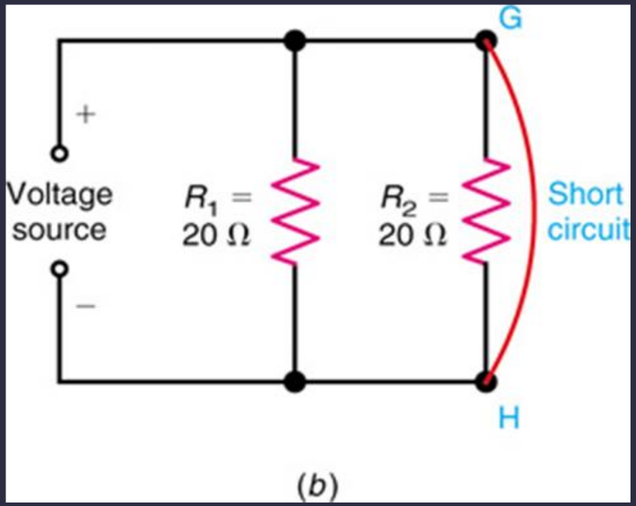
Shorts in a Parallel Circuit.

- A short circuit has zero resistance, resulting in excessive current in the shorted branch.
- A shorted branch shorts the entire circuit.
- Current does not flow in the branches that are not shorted. They are bypassed by the short circuit path that has zero resistance.

Troubleshooting: Opens and Shorts in Parallel Circuits

- A Short in a Parallel Circuit.





The other branches are shorted out. The total current is very high.

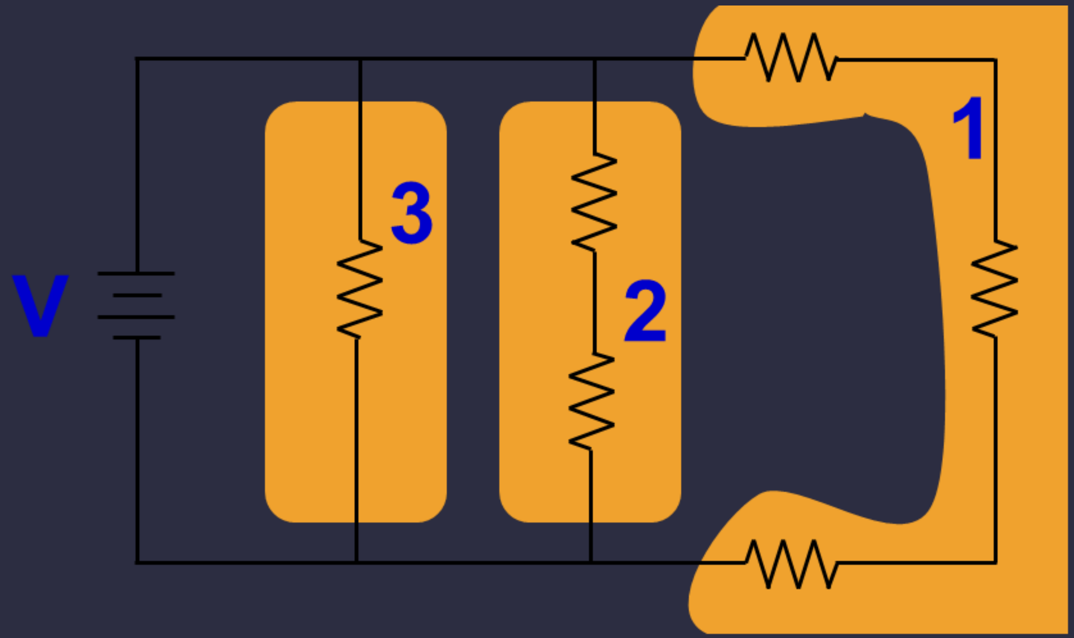
Finding R_T for Series-Parallel Resistances

Overview of Series-Parallel Circuits.

- A **series-parallel circuit**, or **combination circuit**, combines both series and parallel connections.
- Most electronic circuits fall into this category.
  Series-parallel circuits are typically used when different voltage and current values are required from the same voltage source.
- Series components form a **series string**.
- Parallel components form a **parallel bank**.

Finding R_T for Series-Parallel Resistances

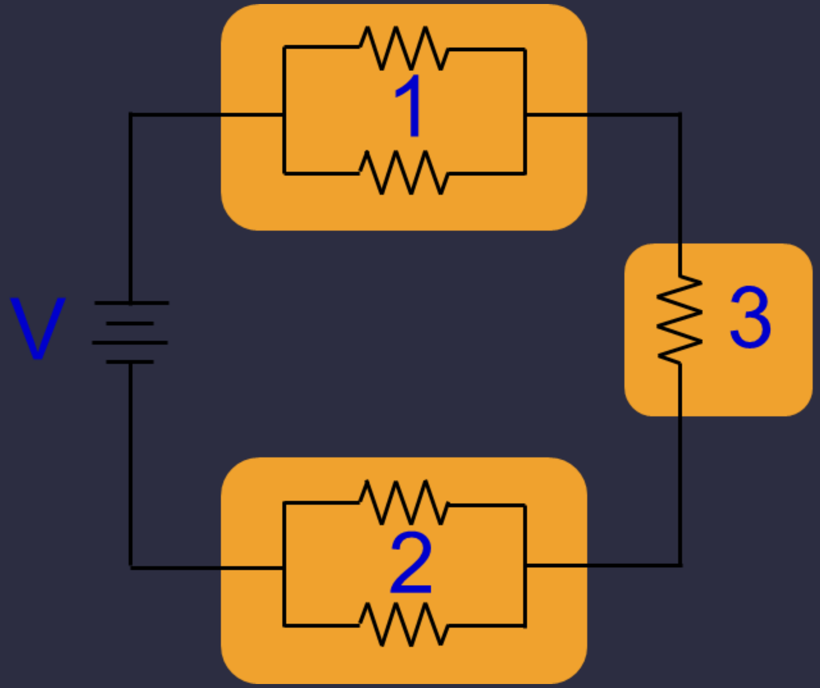
- Overview of Series-Parallel Circuits.



There are three branches in this circuit; sections 1 and 2 are series strings.

Finding R_T for Series-Parallel Resistances

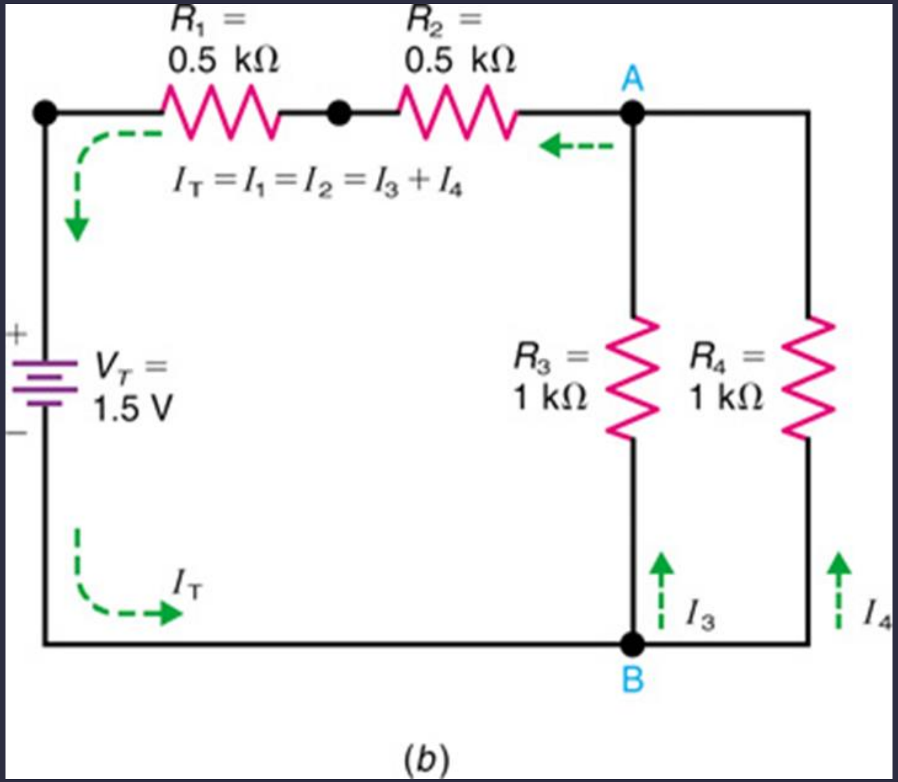
- Overview of Series-Parallel Circuits.



There are three series sections in this circuit; sections 1 and 2 are parallel banks.

Finding R_T for Series-Parallel Resistances

- To find R_T for a series-parallel circuit, add the series resistances and combine the parallel resistances.
- In this diagram, R_1 and R_2 are in series, and R_3 and R_4 are in parallel.
- However, R_2 is <u>not</u> in series with either R_3 or R_4. Resistances in series have the same current, but the current in R_2 is equal to the <u>sum</u> of the branch currents I_3 and I_4.



Schematic diagram of a series-parallel circuit.

Finding R_T for Series-Parallel Resistances

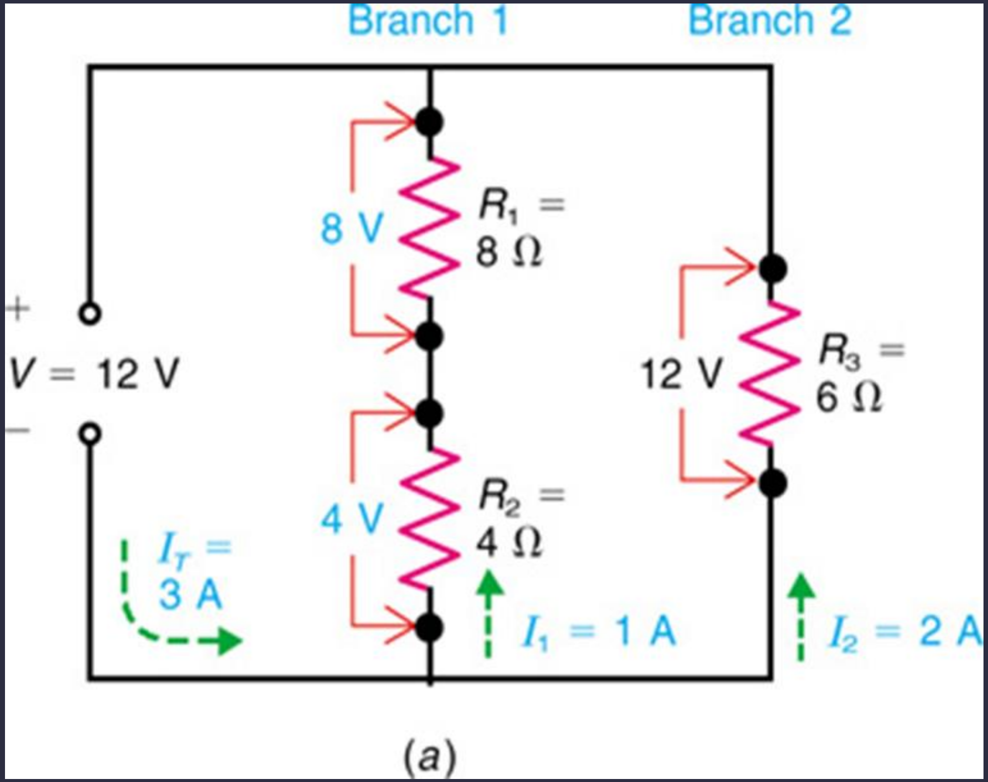
- The series resistances are:

$$0.5\text{k}\Omega + 0.5\text{k}\Omega = 1\text{k}\Omega$$

- The equivalent resistance of the parallel resistances is:

$$\frac{1\ \text{k}\Omega}{2} = 0.5\text{k}\Omega$$

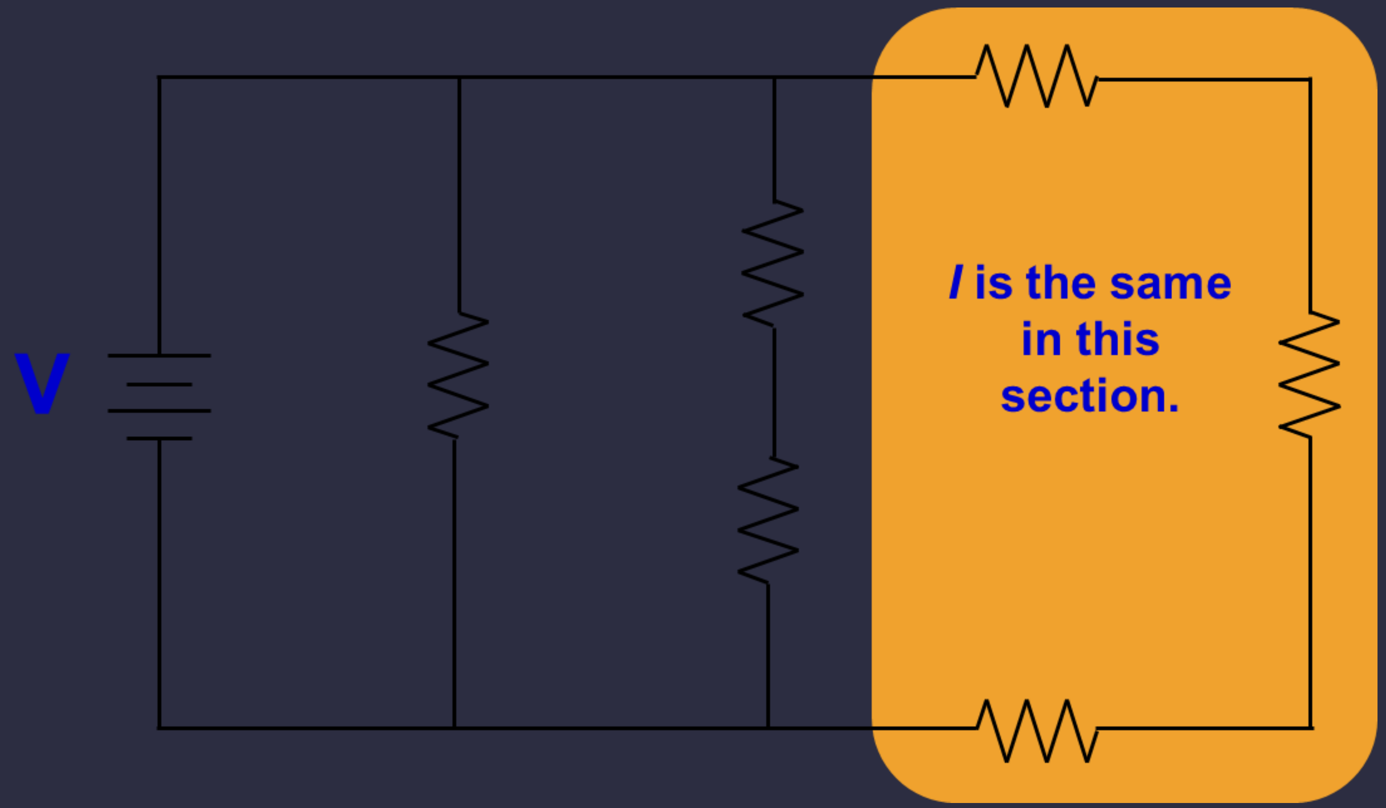
Resistance Strings in Parallel



- In this figure, branch 1 has two resistances in series; branch 2 has only one resistance.

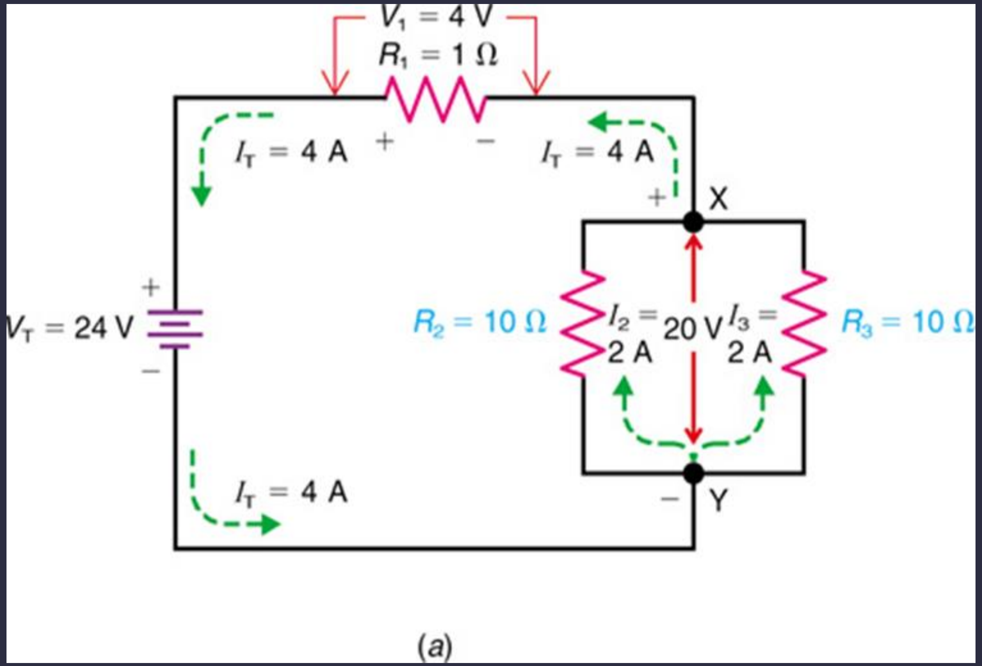
Resistance Strings in Parallel

V is the same across each parallel branch.



Resistance Banks in Series

- In this figure, R_2 and R_3 are parallel resistances in a bank. The parallel bank is in series with R_1.
- There may be more than two parallel resistances in a bank, and any number of banks in series.
- Ohm's Law is applied to the series and parallel components as seen previously.



Parallel bank of R_2 and R_3 in series with R_1

Resistance Banks in Series

- To find the total resistance of this type of circuit, combine the parallel resistances in each bank and add the series resistances.

$$R = \frac{V}{I}$$

$$6\Omega = \frac{10\Omega(\text{of } R_2 + R_3)}{2 \text{ branches}} + 1\Omega(R_1)$$

$$R = \frac{24V}{4A}$$

$$6\Omega = \frac{24V}{4A}$$

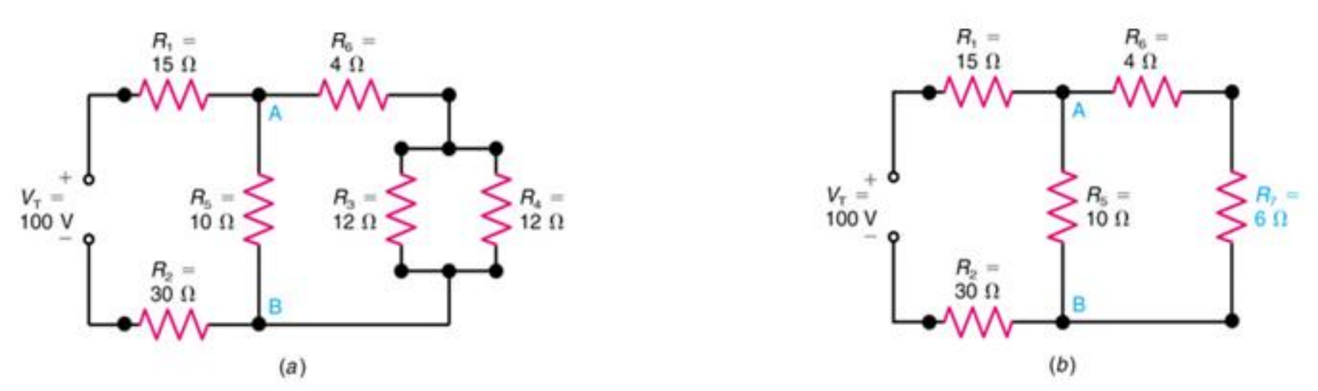
$$6\Omega = 5\Omega + 1\Omega$$

Resistance Banks and Strings in Series-Parallel 2

Example:

- Find all currents and voltages in Figure
  - Step 1: Find R_T.
- Step 2: Calculate main line current as

$$I_T = \frac{V_T}{R_T}.$$

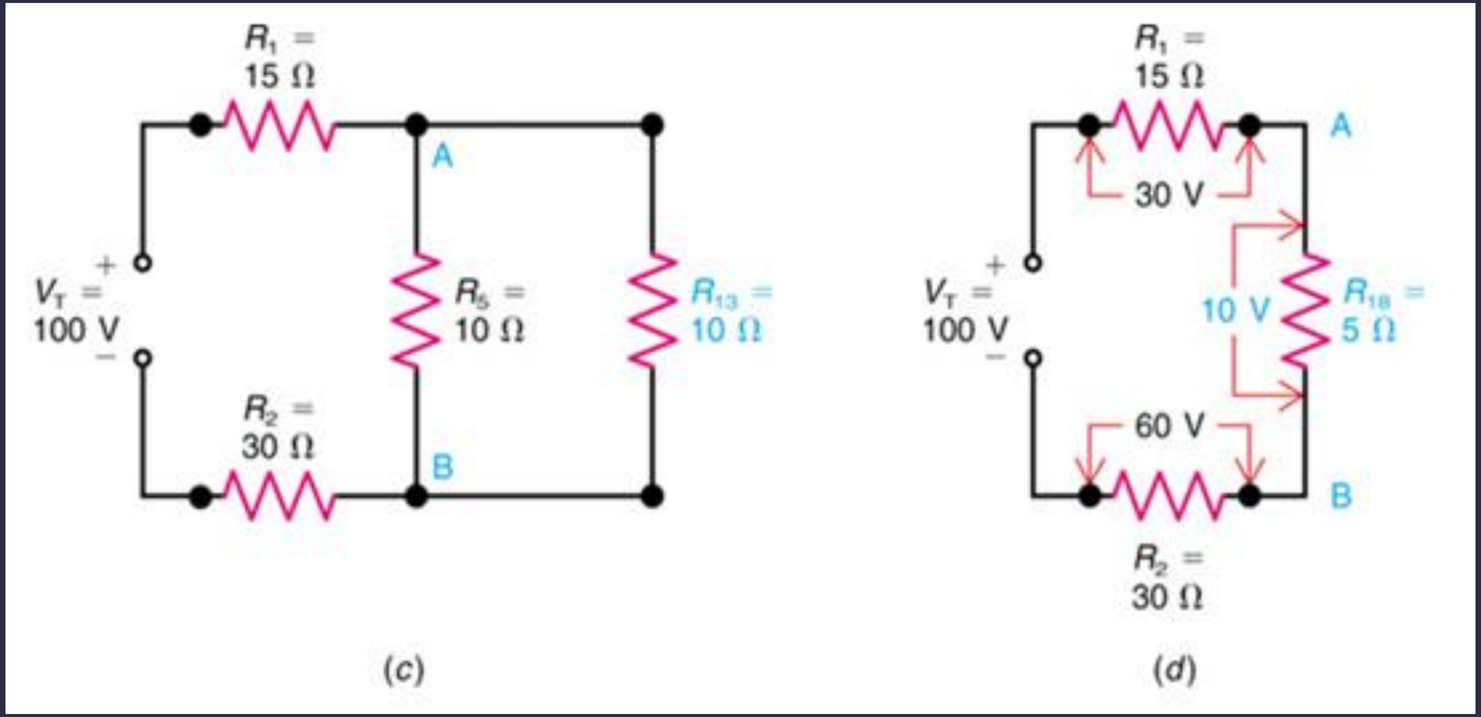


Reducing a series-parallel circuit to an equivalent series circuit to find the R_T.

(a) Actual circuit.

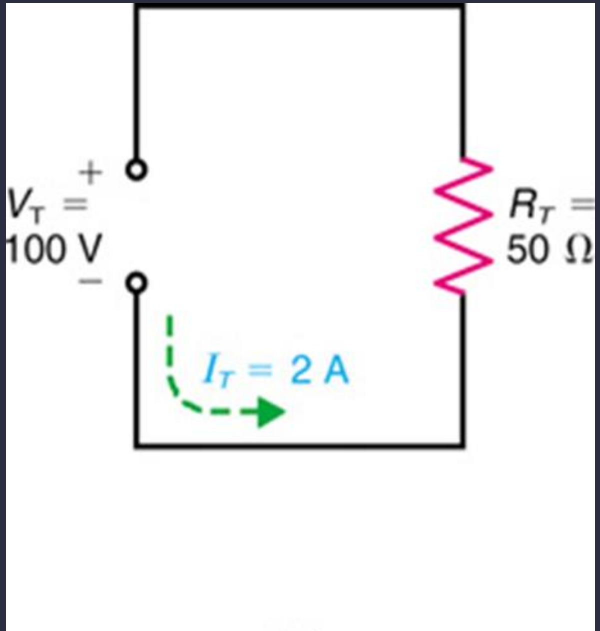
(b) (b) R_3 and R_4 in parallel combined for the equivalent R_7.

Resistance Banks and Strings in Series-Parallel



(c) R_7 and R_6 in series added for R_{13}. (d) R_{13} and R_5 in parallel combined for R_{18}.

Resistance Banks and Strings in Series-Parallel 4



(e)

The R_{18}, R_1, and R_2 in series are added for the total resistance of $50\ \Omega$ for R_T.

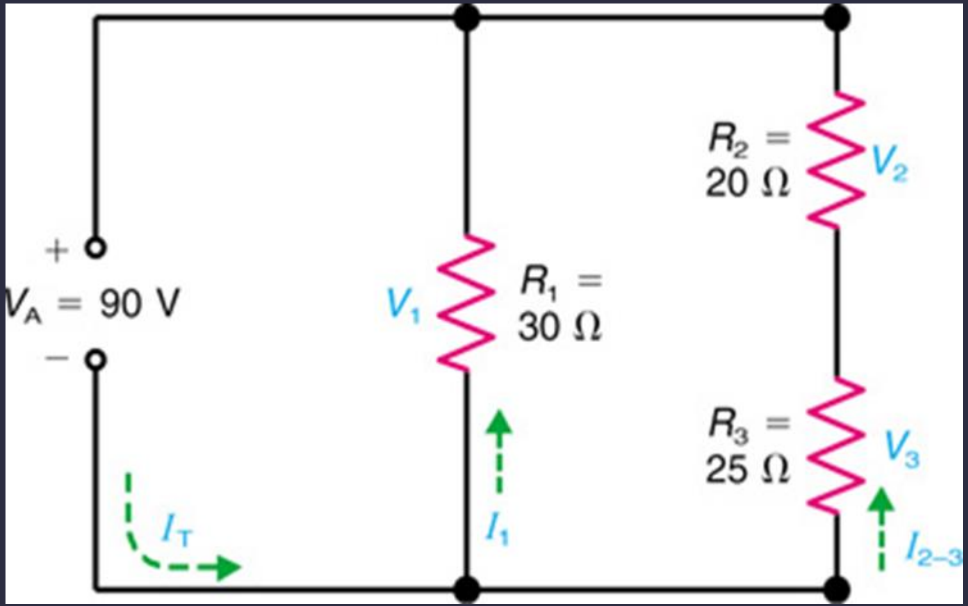
Analyzing Series-Parallel Circuits with Random Unknowns

In solving such circuits, apply the same principles as before:

- Reduce the circuit to its simplest possible form.
- Apply Ohm's Law.

Example:

- In Figure, we can find branch currents I_1 and $I_{2\text{-}3}$, and I_T, and voltage drops V_1, V_2, and V_3, without knowing the value of R_T.



Finding all the currents and voltages by calculating the branch currents first.

Analyzing Series-Parallel Circuits with Random Unknowns

- Find I_1, $I_{2\text{–}3}$, and I_T.

$$I_1 = \frac{V}{R_1}$$

$$I_1 = \frac{90V}{30\Omega}$$

(parallel branches have the same voltage)

$$I_1 = 3A$$

Analyzing Series-Parallel Circuits with Random Unknowns

$$I_{2-3} = \frac{V}{R_{2-3}}$$

$$I_{2-3} = \frac{90V}{20\Omega + 25\Omega}$$

$$I_{2-3} = \frac{90V}{45\Omega}$$

$$I_{2-3} = 2A$$

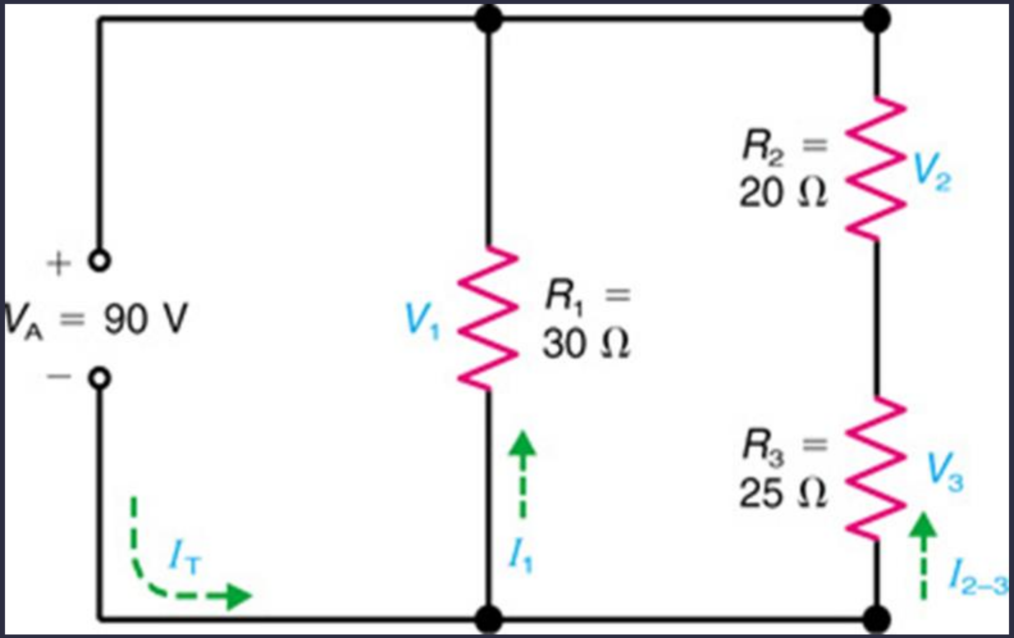
$$I_T = I_1 + I_{2-3}$$

$$I_T = 3A + 2A$$

$$I_T = 5A$$

Analyzing Series-Parallel Circuits with Random Unknowns

- Find voltage drops V_1, V_2, and V_3:



Analyzing Series-Parallel Circuits with Random Unknowns

$V_1 = V_A$

$V_1 = 90\text{V}$

or

$V_1 = I_1 R_1$

$V_1 = 3A \times 30\Omega$

$V_1 = 90\text{V}$

$V_2 = I_{2-3} R_2$

$V_2 = 2A(20\Omega)$

$V_2 = 40\text{V}$

$V_3 = I_{2-3} R_3$

$V_3 = 2A(25\Omega)$

$V_3 = 50\text{V}$

Note: $V_2 + V_3 = V_A$

$40V + 50V = 90V$

Analyzing Series-Parallel Circuits with Random Unknowns

$$R_T = \frac{V_A}{I_T}$$

$$R_T = \frac{90\mathrm{V}}{5A}$$

$$R_T = 18\Omega$$

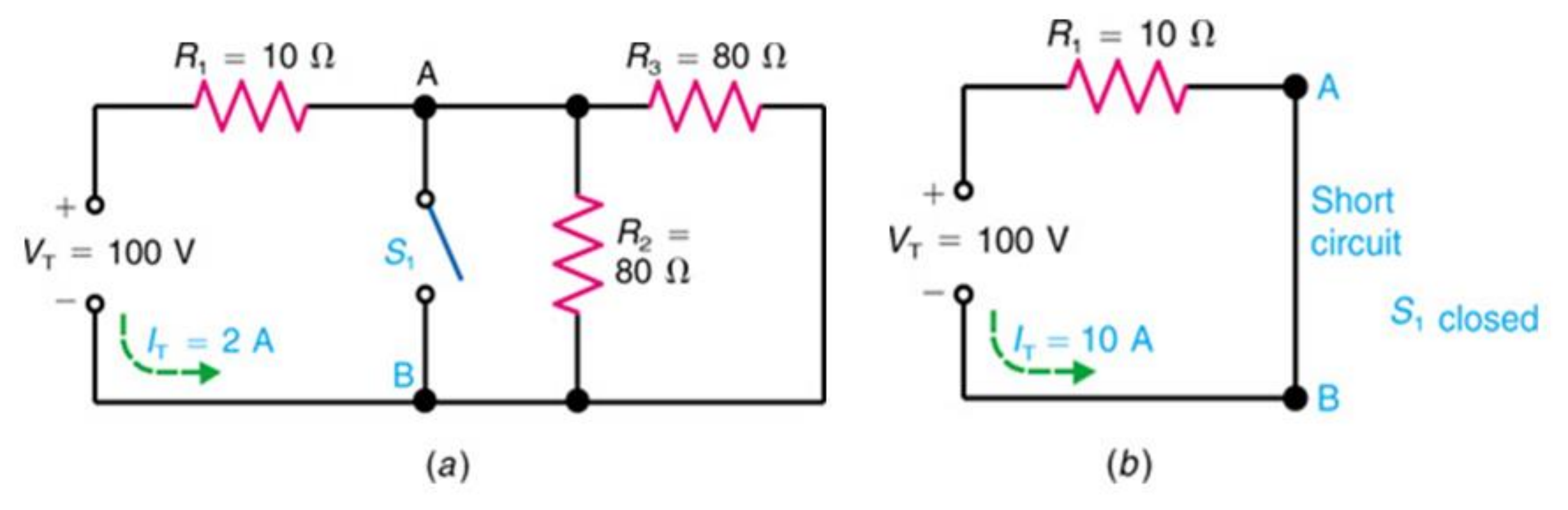
Troubleshooting: Opens and Shorts in Series-Parallel Circuits

- In series-parallel circuits, an open or short in one part of the circuit changes the values in the entire circuit.
- When troubleshooting series-parallel circuits, combine the techniques used when troubleshooting individual series and parallel circuits.

Troubleshooting: Opens and Shorts in Series-Parallel Circuits

Effect of a Short in a Series-Parallel Circuit.

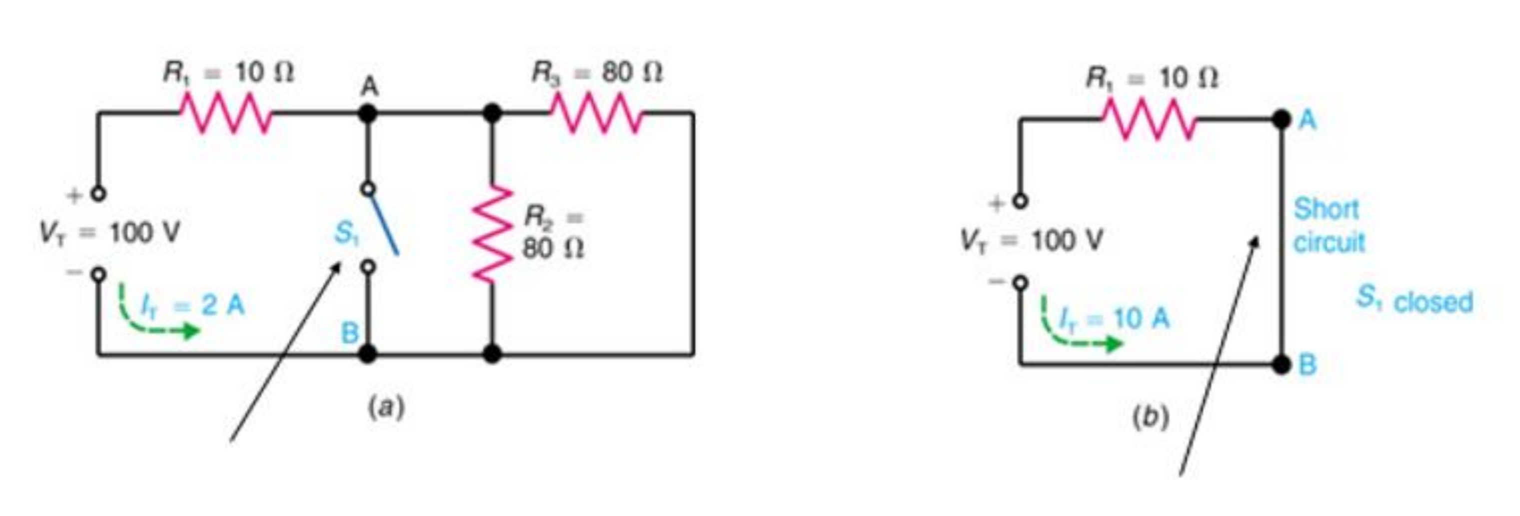
- The total current and total power increase.



Effect of a short circuit with series-parallel connections.
(*a*) Normal circuit with S_1 open.
(*b*) Circuit with short between points A and B when S_1 is closed; now R_2 and R_3 are short-circuited.

Troubleshooting: Opens and Shorts in Series-Parallel Circuits

- Effect of a Short in a Series-Parallel Circuit.

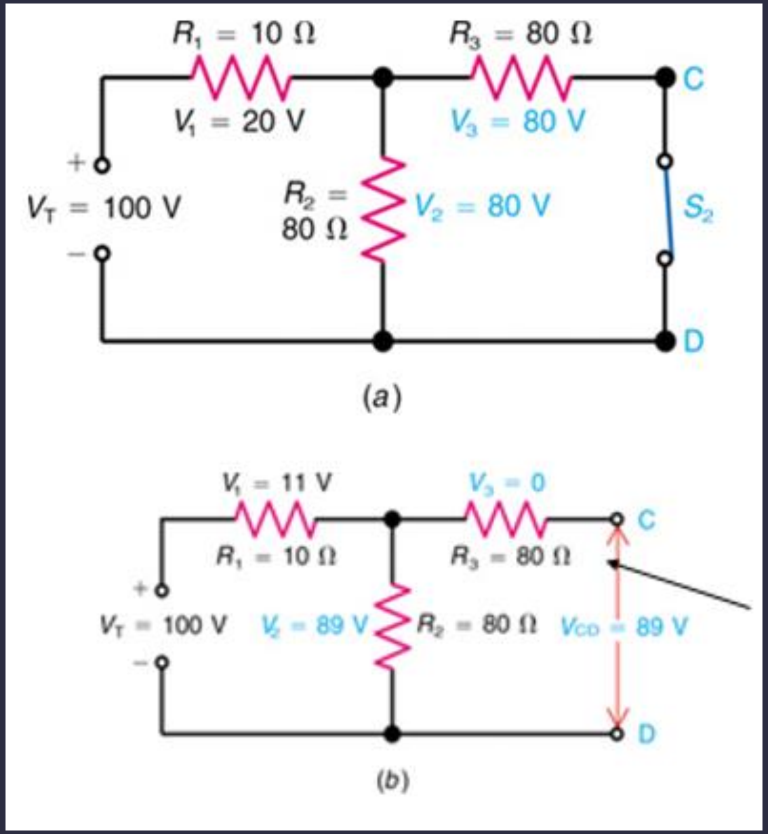


The total current increases from 2A with S_1 open to 10A with S_1 closed.

With S_1 closed, R_2 and R_3 are shorted out.

Troubleshooting: Opens and Shorts in Series-Parallel Circuits

- Effect of an Open in a Series-Parallel Circuit.



Effect of an open path in a series-parallel circuit.

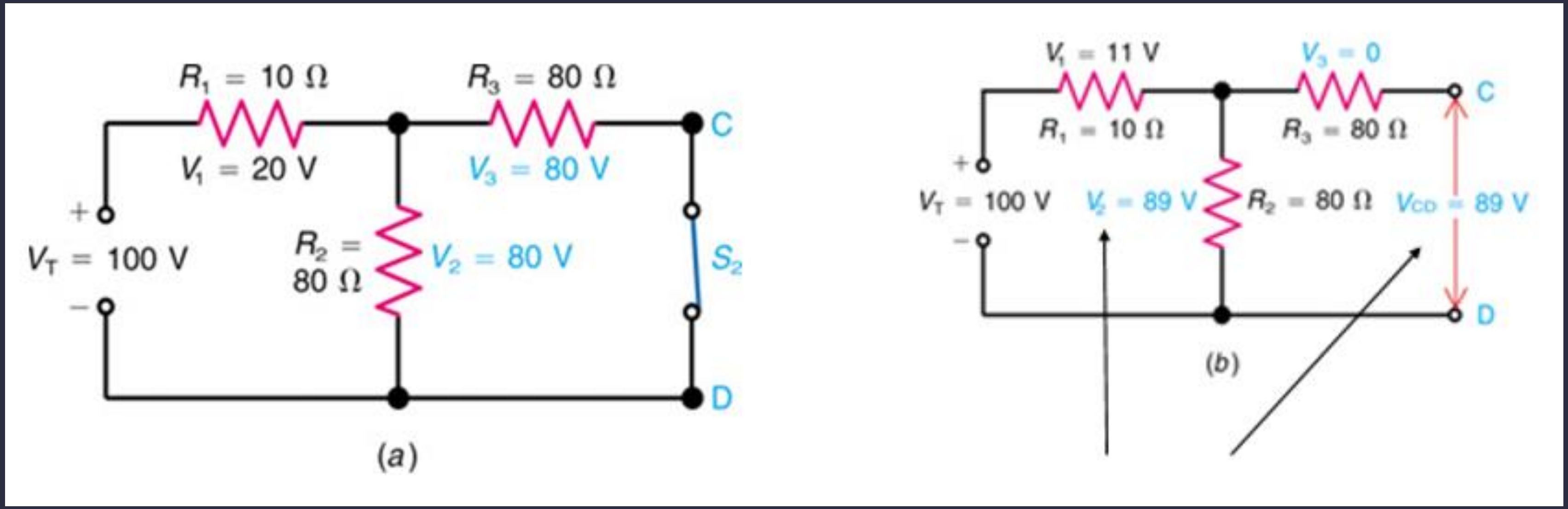
(*a*) Normal circuit with S_2 closed.

(*b*) Series circuit with R_1 and R_2 when S_2 is open. Now R_3 in the open path has no current and zero *IR* voltage drop.

With S_2 open, R_3 is effectively removed from the circuit.

Troubleshooting: Opens and Shorts in Series-Parallel Circuits

- Effect of an Open in a Series-Parallel Circuit.



With S_2 open the voltage across points **C** and **D** equals the voltage across R_2,which is 89V. The voltage across R_3 is zero.